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# SOVIET PHYSICS

## Acoustics

VOLUME 2 NUMBER 4

A Translation

*of the*

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*of the*

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# SOVIET PHYSICS

## Acoustics

*A translation of the "Journal of Acoustics" of the USSR.*

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SOVIET PHYSICS

Acoustics

SOVIET PHYSICS — ACOUSTICS VOLUME 2, NUMBER 4

# RAO'S RULE AND ITS FOUNDATIONS

## SURVEY

**B. B. Kudriavtsev**

In 1940 Rao [1], making use of an empirically established relationship between the temperature coefficients of sound velocity in liquids and the volume temperature coefficients of these liquids, formulated a rule which carries his name. In accordance with Rao's rule, the product of the molecular volume of the liquid $v_m$ and the cube root of the velocity of sound for the given liquid remains constant when the temperature varies:

$$v_m \cdot c^{1/3} = R. \tag{1}$$

The quantity R which is independent of temperature has been given the name of the molecular velocity of sound.* As further investigations of many different liquids has shown, the quantity R retains a constant value over a very wide range of temperatures [2]. From the viewpoint of the investigator of both the acoustical and physicochemical properties of various liquids, R is of considerable interest. It was discovered [3, 4, 5] that the quantity R is additive with respect to the composition of a molecule. Within the boundaries of a given homologous series the molecular velocity of sound increases linearily with an increase in molecular weight; it varies in accordance with the equation

$$R = \alpha M + \beta. \tag{2}$$

As experiment shows, the constant $\alpha$ in Equation (2) has approximately the same value for all homologous series [6], and therefore if Equation (2) is rewritten in the form

$$R/M = c^{1/3}/\rho = \alpha + \beta \cdot 1/M, \tag{3}$$

it follows that we obtain a family of straight lines which diverge from one point (Fig. 1) on the graph which depicts the dependence of $c^{1/3}/\rho$ upon $1/M$ for compounds which belong to various homologous series. The average value of $\alpha$ for various organic compounds is equal to 13.56.

The additivity of R with respect to molecular composition permits definite increments of molecular sound velocity to be ascribed to atoms of separate elements which are included in a molecule of an organic compound, and it permits us to compute R for various compounds by adding the corresponding increments on the basis of the composition of the molecule. It was also established that the molecular sound velocity is, along with the special features indicated above, a constitutive property and therefore can be computed as an additive function of the bonds which exist in a given compound [4]. Making use of the increments of the bonds, it is possible to compute the molecular sound velocities in a manner which is in satisfactory agreement with experimental results on the basis of the corresponding structural formulas of organic compounds. Thus, for example, in the case of saturated hydrocarbons it is necessary to make use of the equation

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* The quantity R has the dimensions of $cm^{10/3}\ sec^{-1/3}$, and therefore the name "molecular velocity of sound" must be considered a poor one.

$$R = 4.25(n-1) + 95.2(2n+2), \tag{4}$$

for computing R, where (2n + 2) is the number of (C—H) bonds and (n−1) is the number of (C—C) bonds.

In making computations of this type a small discrepancy is usually observed between the computed and experimentally obtained values of R; this discrepancy is explained by the effect of the molecule as a whole upon the magnitude of the increments of individual bonds. The fluctuations in the values of the increments are especially great when a transition is made from one molecule to another in the case of bonds which bind halogen atoms together [7]. For members of a given homologous series the molecular sound velocity is associated with other additive properties of the substance by a linear function (for example, with such properties as the parachor, the constant b in the Van der Waals equation, the molecular magnetic rotation, the critical volume and with certain other physicochemical properties [5]). It has been empirically found [5] that the boiling point of the substance is a linear function of the logarithm of R:

$$t_{\text{boil}} = A + B \log R. \tag{5}$$

The constancy of the quantities A and B is maintained within the limits of a given homologous series. Deviations from this rule are observed for associated liquids.

The special features of the molecular velocity of sound have led to numerous attempts [2, 8, 9] at utilizing this quantity in various physicochemical investigations of liquids. In particular, the constitutive nature of R made it possible to utilize acoustical measurements in studying the structure of molecules [10, 11].

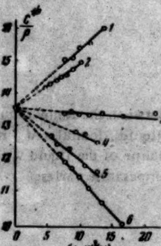

Fig. 1. The dependence of the quantity $c^{1/3}/\rho$ upon the quantity which is the inverse of the molecular weight 1/M: 1) Aliphatic hydrocarbons, 2) olefins, 3) aliphatic alcohols, 4) aromatic hydrocarbons, 5) halogen derivatives of fatty hydrocarbons, 6) fatty acids.

The above-cited deviation ΔR from additivity can be associated with the structural peculiarities of the corresponding molecules when computing the molecular velocity of sound by means of the increments of R.

The linear relationship that exists between the molecular sound velocity R and the molecular refraction, molecular viscosity, molecular magnetic rotation, parachor, etc., within the boundaries of a given homologous series makes it possible to utilize acoustical measurements in order to determine molecular weights [12, 13]. For this purpose several relationships have been proposed. One of them, for example, is

$$M = \frac{B\rho}{c^{1/3} - A\,\dfrac{n^2-1}{n^2+2}}, \tag{6}$$

where c is the velocity of sound, $\rho$ is the density, n is the index of refraction, A and B are the empirically determined constants which retain an invariant value within the limits of one homologous series.

It has been experimentally shown that the molecular weights computed on the basis of acoustical data agree satisfactorily with the values obtained by other methods. This made it possible to propose [13] that an acoustical method for determining the molecular weight could be used in the capacity of a method for continuous production monitoring.

In certain liquids the molecular sound velocity is insignificant but varies regularly with a variation in temperature. The proposal has been advanced that this is explained by the variation of the degree of association of the liquid when the temperature varies [14]; on this basis it has been proposed that the following relationship be assumed as a measure of the relative association of the liquid:

$$\frac{c_0^{1/3}\rho_0^{-1}}{c_t^{1/3}\rho_t^{-1}} = R_{\mathrm{rel}}, \tag{7}$$

where $c_0$ and $\rho_0$ are respectively the sound velocity and the density at a certain fixed temperature (for example, at 0° C), and $c_t$ and $\rho_t$ are these same quantities at the experimental temperature $t$. However, such computations must be approached very critically. As we shall show below, investigations which have been conducted in recent years [15, 16] have shown that the molecular sound velocity is insensitive to the appearance of even relatively stable chemical compounds in a liquid medium. Therefore there is no basis for thinking that the molecular assoication of liquids which is caused by the relatively weak Van der Waals forces will affect the magnitude of R and its behavior.

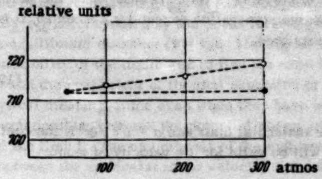

Fig. 2. Molecular sound velocity in normal heptane at 20° as a function of pressure.

For a long time Rao's rule did not have a real theoretical basis; however, at the present time several proofs of this interesting mechanism have been proposed [17, 18, 19, 20, 21].

Before going on to an analysis of the proofs of Rao's rule, we should once more precisely determine its physical meaning. At first glance Eq. (1) correlates the velocity of sound $c$ in the liquid with the molecular volume $v_m$ of the latter. However, in fact this is not so, and Rao's rule actually correlates the variations of the sound velocity and the volume of the liquid with the temperature variation. It is possible to clarify the meaning of the indicated difference by examining the graph in Fig. 2, in which the values of pressure are plotted along the axis of abscissas and the product $c^{1/3}\cdot v_m$ for normal heptane at 20° [22] are plotted along the axis of ordinates. Here the black dots refer to the case when the volume of the liquid was varied as a result of temperature variations, and the open dots refer to experiments which were conducted under isothermal conditions. In the latter case the variation in the volume was achieved by increasing the external pressure. As is obvious from the graph, Eq. (1) is not fulfilled for isothermal variations in the volume.

One of the first proofs of Rao's rule [23] was based upon the concept of simulating the structure of the liquid. It is possible to assume [24] that an ideal liquid differs from a solid body in that there are no shear moduli. In that case it is possible to obtain an expression for the sound velocity $c$ in which the second derivative of the potential energy of interaction between molecules $\Phi(r)$ with respect to distance is included along with the intermolecular distance $r$ – namely:

$$c = \sqrt{\Phi''(r)/\rho\cdot r}, \tag{8}$$

where $\rho$ is the density of the liquid.

If the potential energy of the molecular interaction is approximately expressed by an equation of the form

$$\Phi(r) = C\cdot r^{-n_1}, \tag{9}$$

where C and $n_1$ are constants and $r$ is the distance between the centers of the molecules, then we will obtain the following expression for the velocity of sound after substituting $K\cdot\rho^{-1/3}$ for $r$:

$$c^2 = D\rho^{\frac{n_1}{3}}, \tag{10}$$

where D is a certain constant.

It can easily be seen that Rao's rule will be fulfilled in the case where $n_1$ is equal to 18 [23].

Usually a two-term equation of the type

$$\Phi(r) = A_1/r^{\nu} - B_1/r^{\mu}, \tag{11}$$

[25, 26, 27] is used in the capacity of an expression for the potential energy of molecular interaction; in this equation the first term is due to repelling forces and the second term is due to attractive forces. The representation of the potential energy of the liquid in the form of a two-term expression of the type represented by Eq. (11) is used by Nozdrev and Lependin [17] and by Tarasov [18] in order to prove Rao's rule. Drawing upon the similarity between a liquid and a three-dimensional quasi-crystal as far as the propagation of sound is concerned, it is possible to write the following equation for the characteristic temperature:

$$\theta = \frac{h}{2\pi k}\sqrt{\sigma/m}, \tag{12}$$

in which h is Planck's constant, k is Boltzmann's constant, m is the molecular mass and $\sigma = \partial^2\Phi/\partial r^2$ is the coefficient of quasi-elastic force. In this case the following equation will be valid for the velocity of sound:

$$c = \theta v_m^{1/3}\, 2\frac{k}{h} N^{-1/3}. \tag{13}$$

Here $v_m$ is the molecular volume, N is Avogadro's number.

Under the assumptions which have been made above, the equation for Rao's rule will acquire the form:

$$\frac{\frac{1}{c}\frac{dc}{dt}}{\frac{1}{v_m}\frac{dv_m}{dt}} = \frac{d\ln c}{d\ln v_m} = \frac{d\ln\theta}{d\ln v_m} + \frac{1}{3}. \tag{14}$$

Substituting the value of $\sigma$ [computed on the assumption that $\Phi(r)$ can be expressed in terms of Eq. (11)] into Eq. (12) and performing the appropriate transformations, we find that

$$\theta = \frac{h}{2\pi k}\sqrt{\frac{2\nu(\nu-\mu)}{\mu}}\, A_1 \frac{1}{r_0^{(\nu+2)/2}}. \tag{15}$$

Replacing the molecular volume by its value $v_m = Nr^3$, we obtain

$$\frac{d\ln\theta}{d\ln v_m} = -\frac{\nu+2}{6}, \tag{16}$$

whence, taking Relationship (14) into account, we have

$$\frac{\frac{1}{c}\frac{dc}{dt}}{\frac{1}{v_m}\frac{dv_m}{dt}} = -\frac{\nu}{6}. \tag{17}$$

Thus Rao's rule will evidently be fulfilled when $\nu = 18$. In the above derivation it was assumed that the coordination number of the liquids remains invariant when the temperature varies, and that in addition the interaction of only the closest molecules is taken into account. An analogous derivation can be made without these limitations if the interaction of the second and third rows of molecules is taken into account. In that case [27] the equation of state which is obtained on the basis that the energy of molecular interaction can be expressed by Relationship (11) becomes very complex:

$$p = \frac{NkT}{v}\left[1 - \frac{12\varepsilon_M}{kT}\left\{2.4090\left(\frac{v_0}{v}\right)^2 - 2.0219\left(\frac{v_0}{v}\right)^4\right\} + \right. \tag{18}$$
$$\left. + 4\cdot\frac{12\varepsilon_M}{kT}\left\{\left(\frac{v_0}{v}\right)^4\frac{g_L}{g} - \left(\frac{v_0}{v}\right)^2\frac{g_M}{g}\right\}\right],$$

where p is the pressure, v is the volume, N is Avogadro's number, k is Boltzmann's constant, $v_0$ is the equilibrium volume of the liquid, and g, $g_L$, $g_M$ are complex functions of the absolute temperature and the ratio $(v_0/v)$. The quantity $\epsilon_M$ is determined by the potential energy of interaction in the case when the molecules of the liquid are at the equilibrium distances $(v = v_0)$. Making use of [18], it is possible to obtain the expression for the adiabatic compressibility of the liquid and to find the ratio between the temperature coefficient of the sound velocity in the liquid and the coefficient of thermal expansion of the liquid [19]. Such extremely complex computations are devoid of physical meaning, since even when they have been completed it is still necessary to resort to coarse approximate relationships between the potential energy of molecular interaction of the liquid and the other characteristics of the liquid. The end product of the indicated line of reasoning is the establishment of the following relationship between the molecular sound velocity R and the critical parameters of the substance:

$$R = b\,(\gamma T_{\text{к}}/M)^{1/6} v_{\text{к}}, \tag{19}$$

where $\gamma = c_p/c_v$, M is the molecular weight, $T_k$ and $v_k$ are respectively the critical temperature and volume, b is a constant which is equal to 4.44 according to theoretical computation.

The average experimental value of b for three liquified gases and thirteen organic liquids was b = 2.53, whereas for eleven other investigated liquids b - 2.56 [28].

Equation (19) was first obtained by Nozdrev and Lependin [17] on the basis of the simple equation of state for the liquid:

$$p = \frac{kT}{v} + \frac{\partial\Phi(v)}{\partial v}, \tag{20}$$

where p is the pressure, k is Boltzmann's constant, v is the volume, T is the absolute temperature and $\Phi(v) = Av^{-n} - Bv^{-m}$ is the potential energy of molecular interaction. Noting that the velocity of sound is equal to

$$c = \left\{\gamma\left(\frac{\partial p}{\partial \rho}\right)_T\right\}^{1/2}, \tag{21}$$

and assuming that n = 6, it can be shown that for the temperatures given $\theta \leq 0.65$ [i. e., the expression

$$R = A_0\left[\frac{\gamma R T_{\text{к}}}{M}\right]^{1/6} v_{\text{к}}, \tag{22}$$

for the molecular sound velocity is valid approximately up to the boiling point in a manner analogous to Expression (19)]. Attention should be paid to the fact that Eq. (19) was obtained by assuming that n = 4, and Eq. (22) was obtained on the basis of assuming that n = 6. This agrees completely with the assertion made in [29] to the effect that the properties of a liquid substance can be successfully described by making use of various values of the exponent n in Eq. (11). From Eq. (22) it follows that R must vary with temperature in proportion to $\gamma^{1/6}$. The additive properties of R are associated with the additive properties of the critical volume. In a similar manner it is possible theoretically to obtain the experimentally known proportionality between the molecular sound velocity and the parachor.

V. F. Nozdrev [30] made use of the law of corresponding states in order to prove Rao's rule. As we know [31], in order to satisfy the law of corresponding states it is necessary and sufficient that the potential function $\Phi(r)$ for the substance may be written in the form

$$\Phi(r) = \varkappa g(r/\sigma), \tag{23}$$

where g is a function which is identical for all substances and which obeys the law of corresponding states, while the constants $\kappa$ and $\sigma$ can vary when the transition is made from one substance to another. Equation (11) satisfies the formulated condition for any values of A and B provided that $\nu$ and $\mu$ are constant. For liquids which obey the law of corresponding states the relationship

$$Mc^2/T_k = f(\theta) \tag{24}$$

is valid; in this expression $f(\theta)$ is a function of the normalized temperature $\theta = T/T_k$, T is the absolute temperature, $T_k$ is the critical temperature of the substance on the absolute temperature scale. Having made use of Eq. (24), it is possible to obtain an expression for R which is analogous to (22) – namely:

$$R = B^{1/6} v_{\text{к}} (T_{\text{к}}/M)^{1/6}, \tag{24a}$$

where B is a certain quantity which is constant for all substances. For many liquids the multiplier $(T_k/M)^{1/2}$ has approximately the same magnitude, and the absolute value of R is basically determined by the value of the critical volume $v_k$. This latter fact is proven by the numerous examples of substances which have very nearly identical values of sound velocity and at the same time have sharply differing values of molecular sound velocity. The table compares the sound velocities in certain liquids (these velocities were measured for identical normalized temperatures) and the corresponding values of molecular sound velocity. It is easy to see that for one and the same value of c appreciable fluctuations in R are observed. In the last column of the table the ratios between the molecular sound velocity R and the corresponding value of the molecular volume of the liquid $v_m$ are given. It is noticeable that the ratio for different liquids is approximately of the same value.

As we know [32], a somewhat different determination of the normalized temperature $\theta_1$ is utilized successfully for liquids; according to this method

TABLE

| Compound | $t°$ | $\theta$ | c, m/sec | $R$ | $R/v_m$ |
|---|---|---|---|---|---|
| Ethyl alcohol | 30 | 0.124 | 1127 | 614 | 0.95 |
| Isopropyl alcohol | 30 | 0.128 | 1125 | 803 | 1.05 |
| Benzene | 30 | 0.108 | 1276 | 975 | 1.10 |
| h-amyl alcohol | 30 | 0.108 | 1254 | 1175 | 1.10 |
| t-amyl alcohol | 30 | 0.110 | 1180 | 1160 | 1.09 |
| Allyl alcohol | 30 | 0.110 | 1215 | 735 | 1.08 |

$$\theta_1 = \frac{T - T_f}{T_c - T_f}\,, \tag{25}$$

where T is the experimental temperature, $T_f$ and $T_k$ are respectively the temperature of fusion and the critical temperature of the substance. If we introduce the concept of a normalized sound velocity $c_{norm} = c/c_f$, where $c_f$ is the velocity of sound in the liquid at the fusion temperature of the substance, then (as experimental data [2] shows) $c_{norm}$ is a linear function of $\theta_1$, and this function is approximately identical for many liquids. This same type of linear function is also observed for the molecular volumes. For small normalized temperatures the following relationships are approximately fulfilled:

$$c = c_f(1-0.92\,\theta_1),\ v = v_f(1+0.29\,\theta_1). \tag{26}$$

Substituting these values in the expression

$$R = c^{1/n}\,v_m, \tag{27}$$

it can be shown [13] that R will not depend upon temperature if n = 3.17 (i. e., we again obtain Rao's rule). As a first approximation, Rao's rule also derives from the theory of liquid state, which takes into account the presence of a free volume in the liquid. Assuming [33] that the velocity of sound in a liquid is a combination of the transfer of the sound pulse during the intervals between molecular collisions (over the length of the free path) with the velocity of sound in a gas, and the practically instantaneous transmission of the pulse over a distance equal to the diameter of a molecule when molecular collisions take place, it is possible to obtain the following expression for the velocity of sound in the liquid:

$$c = \left[\gamma\,\frac{RT}{M}\right]^{1/2}\left[\frac{v}{v_f}\right]^{1/3}, \tag{28}$$

in which v is the molecular volume of the liquid, $v_f$ is the so-called free volume. Computing the free volume of the liquid in the usual manner [34], we find [20] the following relationship for the molecular velocity of sound:

$$R = v\,[E\gamma^{1/2}]^{1/3}\,[b^{1/3}(MRT)^{1/6}]^{-1}, \tag{29}$$

where E is the energy of vaporization, b is a coefficient which takes into account the packing of the molecules and which is equal to 2 for many liquids. As experiments show, the right-hand side of Eq. (29) does not depend upon temperature for nonassociated liquids.

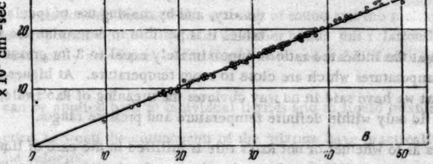

Fig. 3. Dependence of the product of the velocity of sound and the molecular volume (axis of ordinates) upon the intrinsic volume B of the molecules (axis of abscissas).

Schaafs [21] proves Rao's rule in a somewhat different manner; he bases his line of reasoning upon the empirical proportionality which he established between the product of the velocity of sound and the molecular volume of the liquid $(c \cdot v)$, and the intrinsic volume B of the molecules which is computed per mole of liquid –

$$c \cdot v = W \cdot B. \tag{30}$$

Fig. 3 proves the fact that the empirically established Relationship (30) is well fulfilled. It is easy to notice that the ratio $B/v$ goes to unity when the external pressure is increased; therefore the coefficient W has the physical meaning of a certain limiting value of the velocity of sound $c_\infty$ which obtains for an infinitely large pressure, and Eq. (30) can be rewritten in the form

$$c = c_\infty s \frac{B}{v}, \tag{31}$$

where s is a factor which takes into account the elasticity of the molecular collisions.

In accordance with experimental data, $c_\infty = 1600$ m/sec. Assuming that $B = \frac{4}{3}\pi N a^3$, a is the radius of a molecule, N is Avogadro's number, $v = N\ell^3$, and $\ell$ is the side of elementary cube which encompasses the molecules, it is possible to obtain the following expression for Rao's rule:

$$c^{1/3} v = c_\infty^{1/3} (4\pi s/3)^{1/3} a \ell^2 N,$$

or

$$(c/c_\infty)^{1/3} v = (4\pi s/3)^{1/3} a \ell^2 N. \tag{32}$$

When the temperature is increased $\ell^2$ increases while $s^{1/3}a$ decreases, this guarantees that the quantity $a\ell^2 s^{1/3}$ is independent of temperature and serves as the explanation for the fact that the molecular sound velocity is constant when the temperature varies.

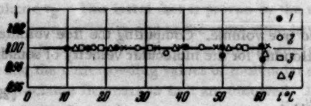

Fig. 4. Molecular sound velocity in solutions of various substances at various temperatures: 1) Camphor in $CCl_4$, 2) diphenylamine in $CCl_4$, 3) benzene-azo-benzene in $CCl_4$, 4) benzene-azo-benzene in benzene. The temperature is shown on the axis of abscissas, and the molecular velocity of sound is shown on the axis of ordinates in relative units.

An examination of the paper by Schaafs convinces us of the fact that his proposed proof of Rao's rule cannot be considered exhaustive, since it is in turn based on an empirical establishment of Relationship (30); this empirical relationship is itself in need of theoretical proof.

In conclusion it should also be said that it is possible to obtain [35] a thermodynamically rigorous expression for the ratio between the temperature coefficient of sound velocity and the temperature coefficient of liquid density, and by making use of special thermodynamic tables it is possible to determine the value of this ratio. Appropriate verification showed that the indicated ratio is approximately equal to 3 for pressures which are close to atmospheric pressure and for temperatures which are close to room temperature. At higher temperatures and pressures this ratio increases. What we have said in no way obviates the meaning of Rao's rule, since (practically speaking) all physical laws are valid only within definite temperature and pressure ranges.

Naturally the question arises as to whether or not Rao's rule is fulfilled in the case of liquid mixtures and solutions. It has been shown experimentally [36] that the molecular sound velocity also remains constant in solutions of various substances when the temperature is varied. What we have said is illustrated in Fig. 4 which depicts the variation of the molecular sound velocity in the various solutions as a function of temperature variation.

Thus the relationship between the temperature coefficient of sound velocity in a liquid and the coefficient of thermal expansion of the latter is a property not only of individual liquids but also of various solutions. In certain solutions the molecular sound velocity is insignificant but varies in a regular manner with temperature (this is similar to what is observed in certain individual liquids). Corresponding computations [36, 37, 15] have shown that for a majority of binary liquids the molecular sound velocity is a quantity which is additive with respect to the composition of the mixture. It is noteworthy that the additive nature of the molecular sound velocities in mixtures can also be extended to apply to mixtures which are characterized by an anomalous variation of sound velocity when the composition of the mixture is varied. In particular, the additive nature of R is still retained when a maximum or minimum is observed on the curves which depict "sound velocity vs. mixture composition." The molecular sound velocity is also additive for mixtures whose components (judging by the fusibility diagrams) form a chemical compound. Figure 5 depicts the variation of R in a phenol-paratoluidine mixture when the concentration of the components of the mixture is varied [16]. The same figure depicts the curve for the sound velocity and the curve for the fusibility of the examined mixture (the presence of a maximum on the latter curve bears witness to the fact that a compound is formed between the components of the mixture). We should underline the linearity of the variation of R as a function of the variation of mixture composition, whereas the analogous function for the velocity of sound passes through a maximum. Additivity is fulfilled still better with respect to composition [15] if the so-called molecular compressibility B is utilized; this quantity is determined [38] by the equation

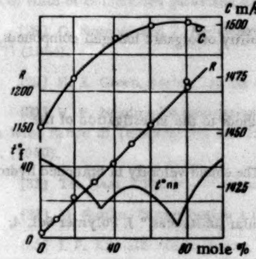

Fig. 5. The dependence of certain physical characteristics of a mixture of phenol-paratoluidine upon the composition of the mixture: c is the velocity of sound, R is the molecular velocity of sound, $t_f$ is the temperature of fusion.

$$B = \frac{M}{\rho}\beta^{1/7}, \tag{33}$$

in which M is the molecular weight, $\rho$ is the density and $\beta$ is the adiabatic compressibility. In the case of mixtures it is assumed that the average molecular weight is used. It is not difficult to show that the following simple relationship obtains between the molecular velocity of sound R and the molecular compressibility B:

$$B = M^{1/7} \cdot R^{6/7}. \tag{34}$$

Therefore the fact that additivity with respect to the mixture composition is fulfilled better when the quantity B is utilized indicates the desirability of maintaining an insignificant variation of the exponents of the quantities which are included in Rao's rule for binary mixtures.

Let us attempt to summarize what we have analyzed above:

1. Rao's rule is a relationship between the velocity of sound and the molecular volume of the liquid, on the one hand, and the temperature of the liquid on the other hand; this relationship can be obtained as a first approximation from practically any theory of the liquid state and is fulfilled only within a definite range of temperatures and pressures.

2. Rao's rule can be applied both to individual liquids and to liquid mixtures.

3. The interaction between the components of the mixture have practically no effect upon the properties of the molecular sound velocity.

4. The additive properties of the molecular sound velocity are due to the additive properties of the molecular volume. The molecular volume determines the absolute magnitude of R.

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# THE DISPERSION EQUATION FOR NORMAL WAVES IN LAMINAR MEDIA*

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**The paper indicates a simple method for obtaining the dispersion equation for waves in laminar media. Several particular cases are examined.**

Let us examine a sound wave radiator which is located in a homogeneous layer that is bounded by two plane-parallel boundaries. Under these conditions a system of so-called "normal waves" is excited in the layer; the amplitudes of these waves decrease as the distance from the source increases, and this attenuation is due to cylindrical divergence, to the leakage of energy through the boundaries of the layer, as well as to the absorption of waves in the medium which fills the layer. The phase velocity and the attenuation of each normal wave are found from the so-called "dispersion" equation which stems from the conditions which obtain on the boundaries of the layer. The author [2] has shown that this equation can be written in the form

$$V_1 V_2 e^{2ikh\cos\vartheta} = 1. \tag{1}$$

Here $V_1 = V_1(\vartheta)$ and $V_2 = V_2(\vartheta)$ are the reflection coefficients of a plane wave from the lower and upper boundaries of the layer respectively, $\vartheta$ is the angle of incidence of the plane wave; $k = \omega/c = 2\pi/\lambda$ is the wave number in the layer, $h$ is the thickness of the layer.

The transcendental Eq. (1) has, in the general case, an infinite series of complex roots $\vartheta = \vartheta(\vartheta_l) = 1, 2, 3, \ldots$. Each root corresponds to a normal wave for which the dependence of the wave amplitude upon the coordinates $x$ and $z$ which are respectively directed parallel and perpendicular to the boundaries of the layer (the origin is assumed to be located on the lower boundary of the layer) is given by the expression

$$\psi_l(x, z) = [V_1(\vartheta_l)\, e^{ikz\cos\vartheta_l} + e^{-ikz\cos\vartheta_l}]\, e^{ikx\sin\vartheta_l} \tag{2}$$

Thus the quantity $k\sin\vartheta_l$ will play the part of the wave number when each normal wave is propagated along the layer. Therefore the phase velocity of a normal wave will be

$$v_l = \frac{\omega}{\mathrm{Re}\,(k\sin\vartheta_l)}, \tag{3}$$

and the attenuation coefficient** will be

$$\beta_l = \mathrm{Im}\,(k\sin\vartheta_l). \tag{4}$$

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** The symbols Re and Im, as always, designate the real and imaginary portions of the corresponding quantity.

The representation of the dispersion equation in the form of Relationship (1) is extremely practical, since no characteristics of the field in the half-spaces which bound the layer on both sides are included in this expression. These half-spaces can also be laminarly inhomogeneous (i. e., the parameters of the media in them can depend upon z in any arbitrary manner). In a number of cases it is possible to make use of approximate methods to determine the coefficients of reflection $V_1$ and $V_2$.

In Section 1 we shall show how Eq. (1) can be obtained in a very simple manner without performing the detailed theoretical analysis which is contained in [1].

In Section 2 an analogous dispersion equation will be obtained for the case of the propagation of elastic waves in a layer which has shear resistance (henceforth we shall call such a layer "hard").

In Section 3 it is shown that if the medium consists of any aggregate of layers, then the propagation velocities and the attenuation of normal waves can be found from Eq. (1) when it is applied to any of the layers.

Equation (1) can be applied to the case where the parameters of the medium vary in a continuous manner when z varies. In that case it is possible to imagine a homogeneous layer of infinitely small thickness at any arbitrary level z; Eq. (1) can be applied to this layer. Since in this case h → 0, Eq. (1) can be written

$$V_1V_2 = 1, \tag{5}$$

where $V_1$ and $V_2$ are the reflection coefficients from the lower and upper half-spaces with respect to the selected level. According to what is proven in Section 3, Eq. (5) yields one and the same values for the characteristics of the normal waves, irrespective of where the indicated level is selected. The skillful selection of the location of of this level can substantially simplify the problem.

## 1. A Simple Method for Obtaining the Dispersion Equation

The waves in the layer can be assumed to be plane at comparatively large distances from the radiator.

Taking into account the fact that as a result of the reflections of waves from the boundaries of the layer it is possible for waves to appear which are propagated both in a positive and in a negative z-axis direction, the field in the layer can be written in the form

$$\left.\begin{array}{c} \varphi = \varphi_+ + \varphi_-, \\ \varphi_+ = Ae^{i(k_z z + k_x x)}, \qquad \varphi_- = Be^{i(-k_z z + k_x x)}, \\ k_x = k\sin\vartheta, \qquad k_z = k\cos\vartheta, \end{array}\right\} \tag{6}$$

where $\vartheta$ is an angle which is an arbitrary one for the time being, and A and B are arbitrary constants. We shall use $\varphi$ to denote the acoustic potentials; $\varphi_+$ represents a wave which is propagated in a direction which makes the angle $\vartheta$ with the positive z-axis direction. Analogously, $\varphi_-$ yields a wave whose direction of propagation makes an angle of $\pi - \vartheta$ with the z-axis.

Expression (6) satisfies the wave equation. Let us now require that this equation satisfy the boundary conditions as well. The latter can be expressed in terms of the reflection coefficients from the bondary of the layer, $V_1$ and $V_2$. In order to do this let us examine Fig. 1. At the lower boundary $\varphi_-$ is the incident wave and $\varphi_+$ is the reflected wave. Therefore the equation

$$(\varphi_+/\varphi_-)_{z=0} = V_1,$$

must be fulfilled, where $V_1$ is the coefficient of reflection from the lower boundary. Substituting the values of $\varphi_+$ and $\varphi_-$ from Eq. (6) into this expression, we find

$$A/B = V_1. \tag{7}$$

At the upper boundary $\varphi_+$ is the incident wave and $\varphi_-$ is the reflected wave. Therefore the following relationship must obtain:

$$(\varphi_-/\varphi_+)_{z=h} = V_2,$$

or, taking (6) into account, we obtain

$$\frac{B}{A} e^{-2ik_z h} = V_2. \tag{8}$$

Multiplying Eq. (7) by Eq. (8) we obtain

$$V_1 V_2 e^{2ik_z h} = 1, \tag{9}$$

which coincides with Eq. (1).

For any $\vartheta = \vartheta_\ell$ (where $\ell = 1, 2, \ldots$ is the solution of the latter equation) Expression (6) for $\varphi$ satisfies both the wave equation and the boundary conditions. Therefore a complete solution will consist of the sum of such expressions for various values of $\ell$.

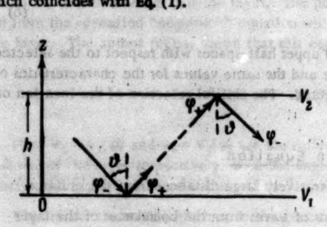

Fig. 1.

Let us examine several particular cases.

a) Let the thickness of the layer h increase without limit. Let us assume that the medium has a certain attenuation (Im k > 0), so that $e^{2ik_z h} \rightarrow 0$. In traversing the path from one boundary to the other waves will be completely attenuated, and normal waves which are formed by means of multiple reflections from both of the layer boundaries will be absent. In that case only surface waves which travel along each of the boundaries will be present. Assuming that h → ∞, we obtain

$$1/V_1 V_2 = 0 \tag{10}$$

from Eq. (9), or taken separately,

$$1/V_1 = 0, \ 1/V_2 = 0. \tag{11}$$

These would be the dispersion equations for the surface waves on each of the boundaries.

b) Let us examine a liquid layer which is bounded from below by a homogeneous elastic half-space with a finite value for its shear modulus, and by a sufficiently rarefied medium from above. The coefficient of reflection for sound waves at the boundary between the liquid and the solid body can be written in the form

$$V_1 = \frac{Z_1 \cos^2 2\gamma_1 + Z_t \sin^2 2\gamma_1 - Z}{Z_1 \cos^2 2\gamma_1 + Z_t \sin^2 2\gamma_1 + Z}, \tag{12}$$

where Z, $Z_1$ and $Z_t$ respectively denote the wave impedances of acoustic waves in the liquid, and of longitudinal and transverse waves in the solid body:

$$Z = \frac{\rho c}{\cos \vartheta}, \quad Z_1 = \frac{\rho_1 c_1}{\cos \vartheta_1}, \quad Z_t = \frac{\rho_1 b_1}{\cos \gamma_1}, \tag{13}$$

where $\rho$ and $\rho_1$ are the densities of the media in the layer and in the lower half-space, c and $c_1$ are velocities of the longitudinal waves, $b_1$ is the velocity of the transverse waves in the lower half-space; $\vartheta$ is the angle of incidence, $\vartheta_1$ and $\gamma_1$ are the angles of refraction for the longitudinal and transverse waves.

Since the upper boundary of the layer is presumed to be free, we have $V_2 = -1$. Now Eq. (9) can be written in the form

$$i \tan k_z h = \frac{1}{Z}(Z_1 \cos^2 2\gamma_1 + Z_t \sin^2 2\gamma_1). \tag{14}$$

We obtain the dispersion equation for a liquid layer which lies on an elastic half-space (such a layer was examined in detail by Sherman [2] in his time).

The equation $1/V_1 = 0$ in the case being investigated is written as follows:

$$Z_1 \cos^2 2\gamma_1 + Z_t \sin^2 2\gamma_1 + Z = 0. \tag{15}$$

This is the dispersion equation for a surface wave on the boundary between the liquid and the solid body. If we introduce the notation

$$r_1 = \left(\frac{b_1}{c}\right)^2, \quad q_1 = \left(\frac{b_1}{c_1}\right)^2, \quad s = \sin^2 \gamma_1, \tag{16}$$

it can also be written in the form

$$4s\sqrt{1-s}\sqrt{q-s} + (1-2s)^2 = -\frac{\rho}{\rho_1}\sqrt{\frac{q-s}{r-s}}. \tag{17}$$

In the case where $\frac{\rho}{\rho_1} \ll 1$ the right-hand side may be neglected, and the equation becomes the well-known equation for a Raleigh wave:

$$4s\sqrt{1-s}\sqrt{q-s} + (1-2s)^2 = 0. \tag{18}$$

c) Making use of Eq. (9) we obtain the dispersion equation for a three-layer liquid medium (Fig. 2). Let us assume that the layers with the parameters $\rho$, c are bounded above by free space. Let us apply Eq. (9) to this layer. In that case $V_2 = -1$, and $V_1$ is the coefficient of reflection from the layer with the thickness d.

In Section 3 it is shown that*

$$V_1 = \frac{R + V_0 e^{2b}}{1 + RV_0 e^{2b}},$$

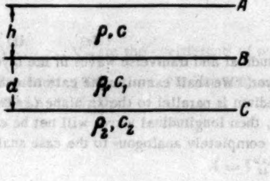

Fig. 2.

where R is the coefficient of reflection from the separation boundary of the layers (the boundary B in Fig. 2), $V_0$ is the coefficient of reflection from the lower boundary of the lower layer (boundary C), and besides this $b = ik_1 d \cos \vartheta_1$, where $k_1 = \omega/c_1$.

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* Here we make use of Formula (43), having somewhat altered and simplified the notation used in it.

Let us introduce the definition

$$Z = \frac{\rho c}{\cos\vartheta}, \quad Z_1 = \frac{\rho_1 c_1}{\cos\vartheta_1}, \quad Z_2 = \frac{\rho_2 c_2}{\cos\vartheta_2}, \quad \alpha_1 = k_1 \cos\vartheta_1,$$

where $\vartheta, \vartheta_1, \vartheta_2$ are the angles which are formed with the z axis in each of the layers by the normal to the wave fronts; here

$$\frac{\sin\vartheta}{c} = \frac{\sin\vartheta_1}{c_1} = \frac{\sin\vartheta_2}{c_2}. \tag{19}$$

Then

$$R = \frac{Z_1 - Z}{Z_1 + Z}, \quad V_0 = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

and the expression for $V_1$ can be written in the form:

$$V_1 = \frac{(Z_1 + Z_2)(Z_1 - Z)\, e^{-i\alpha_1 d} + (Z_2 - Z_1)(Z_1 + Z)\, e^{i\alpha_1 a}}{(Z_1 + Z_2)(Z_1 + Z)\, e^{-i\alpha_1 d} - (Z_2 - Z_1)(Z_1 - Z)\, e^{i\alpha_1 d}}. \tag{20}$$

Thereafter, Eq. (9) can be represented in the following form:

$$\tan \alpha h = -\frac{m_1 \alpha}{\alpha_1} S, \tag{21}$$

where

$$S = \frac{m_1 \alpha_2 \tan(\alpha_1 d) + i m_2 \alpha_1}{m_1 \alpha_2 - i m_2 \alpha_1 \tan(\alpha_1 d)}, \tag{22}$$

$$m_1 = \frac{\rho_1}{\rho}, \quad m_2 = \frac{\rho_2}{\rho}, \quad \alpha = \frac{\omega}{c}\cos\vartheta, \quad \alpha_2 = \frac{\omega}{c_2}\cos\vartheta_2. \tag{23}$$

The dispersion equation for a three-layer medium was obtained in this form in [3].

## 2. Normal Waves in a Solid Layer

An interesting feature of this case is the presence of both longitudinal and transverse waves in the layer; these waves interact between one another on the boundaries of the layer. We shall examine the case of wave polarization which is such that the velocity of the particles of the medium is parallel to the xz plane (a SP-wave). If the velocities are directed perpendicular to this plane (a SH-wave), then longitudinal waves will not be excited when reflections from the boundaries occur and the problem becomes completely analogous to the case analyzed in the preceding section.

The potentials of the longitudinal and transverse waves in the layer (provided that the same assumptions are valid as those which obtain above) can be written in the form $\varphi = \varphi_+ + \varphi_-$ for the potential of longitudinal waves (P-waves), and in the form $\psi = \psi_+ + \psi_-$ for the potential of the transverse waves (S-waves); here

$$\left.\begin{aligned} \varphi_+ &= A e^{ik_z z}, & \varphi_- &= B e^{-ik_z z} \\ \psi_+ &= C e^{i\varkappa_z z}, & \psi_- &= D e^{-i\varkappa_z z} \end{aligned}\right\} \tag{23 •}$$

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• [Two Equations (23) occur in the original – editors' note].

and

$$k_z = k\cos\vartheta, \qquad \varkappa_z = \varkappa\cos\gamma. \tag{24}$$

The angle $\gamma$ is related to the angle $\vartheta$ by the expression

$$k\sin\vartheta = \varkappa\sin\gamma = k_x, \tag{25}$$

where $\underline{k}$ and $\varkappa$ are the wave numbers for longitudinal and transverse waves. In Eq. (23) we omit the factor $e^{ik_x x}$ which is common to all of the expressions.

Let us now turn to the relationships which obtain on the boundaries of the layer.

Let us define the various coefficients of reflection at the lower boundary of the layer $V_{11}^{(1)}$, $V_{21}^{(1)}$, $V_{22}^{(1)}$ and $V_{12}^{(1)}$. $V_{11}^{(1)}$ characterizes the amplitude of the potential of the reflected P-wave when a P-wave is incident; $V_{21}^{(1)}$ characterizes the amplitude of the P-wave when a S-wave is incident; $V_{12}^{(1)}$ characterizes the amplitude of an S-wave when a P-wave is incident; $V_{22}^{(1)}$ characterizes the amplitude of an S-wave when a S-wave is incident. The coefficients $V_{11}^{(1)}$ depend upon the angles of incidence $\vartheta$ and $\gamma$.

At the lower boundary the waves $\varphi_-$ and $\psi_-$ are incident waves and the waves $\varphi_+$ and $\psi_+$ are reflected waves.

The amplitude of the reflected wave $\varphi_+$ will be determined by the amplitudes of both incident waves $\varphi_-$ and $\psi_-$, as well as by the coefficients $V_{11}^{(1)}$ and $V_{21}^{(1)}$. The same must be said with regard to $\psi_+$. In accordance with this we shall have the following relationships on the lower boundary of the layer:

$$z = 0\begin{cases}\varphi_+ = V_{11}^{(1)}\varphi_- + V_{21}^{(1)}\psi_-\\ \psi_+ = V_{12}^{(1)}\varphi_- + V_{22}^{(1)}\psi_-.\end{cases} \tag{26}$$

Conversely, on the upper boundary $\varphi_+$ and $\psi_+$ will be incident waves, and $\varphi_-$ and $\psi_-$ will be reflected waves. Therefore we shall have the analogous formulas:

$$z = h\begin{cases}\varphi_- = V_{11}^{(2)}\varphi_+ + V_{21}^{(2)}\psi_+\\ \psi_- = V_{12}^{(2)}\varphi_+ + V_{22}^{(2)}\psi_+,\end{cases} \tag{27}$$

where $V_{11}^{(2)}, \ldots, V_{22}^{(2)}$ are the coefficients of reflection from the upper boundary.

Substituting Expression (23) into (26) and (27), we obtain a system consisting of four homogeneous equations in terms of A, B, C, D:

$$\left.\begin{aligned} A &= V_{11}^{(1)}B + V_{21}^{(1)}D,\\ C &= V_{12}^{(1)}B + V_{22}^{(1)}D,\\ Be^{-ik_z h} &= V_{11}^{(2)}Ae^{ik_z h} + V_{21}^{(2)}Ce^{i\varkappa_z h},\\ De^{-i\varkappa_z h} &= V_{12}^{(2)}Ae^{ik_z h} + V_{22}^{(2)}Ce^{i\varkappa_z h}.\end{aligned}\right\} \tag{28}$$

The system will have solutions which differ from zero only for the condition that the determinant of the system must be equal to zero. As a result we obtain:

$$\left.\begin{array}{l} e^{i(k_z+\varkappa_z)h}\times[V^{(1)}_{11}V^{(1)}_{22}-V^{(1)}_{21}V^{(1)}_{12}]\,[V^{(2)}_{11}V^{(2)}_{22}-V^{(2)}_{21}V^{(2)}_{12}]- \\ \quad -V^{(1)}_{22}V^{(2)}_{22}e^{i(\varkappa_z-k_z)h}-V^{(1)}_{11}V^{(2)}_{11}e^{-i(\varkappa_z-k_z)h}- \\ \quad -(V^{(1)}_{12}V^{(2)}_{21}+V^{(1)}_{21}V^{(2)}_{12})+e^{-i(\varkappa_z+k_z)h}=0. \end{array}\right\} \qquad (29)$$

This is the dispersion equation which in the case being examined is analogous to Eq. (9).

The phase velocity with which the $\ell$th normal wave is propagated along the layer (and the attenuation coefficient) will again be computed according to Formulas (3) and (4).

If we assume that $V^{(1)}_{12}=V^{(1)}_{21}=V^{(2)}_{12}=V^{(2)}_{21}=0$ in Eq. (29) (this corresponds to the purely hypothetical case of the elimination of the interaction between longitudinal and transverse waves on the boundaries), then Eq. (29) will acquire the form:

$$(1-V^{(1)}_{11}V^{(2)}_{11}e^{2ik_zh})(1-V^{(1)}_{22}V^{(2)}_{22}e^{2i\varkappa_zh})=0,$$

i. e., it will be decomposed into two completely equivalent equations, one of which refers to longitudinal waves and the other to transverse waves.

Finally let us note that in solving Eq. (28) it is possible to express three constants in terms of a fourth (for example, B, C, and D in terms of A). Substituting the constants which have been found in this manner into Eq. (23), we obtain functions which characterize the dependence of the field of any normal wave upon z. Again, one coefficient A remains undetermined for each normal wave; this coefficient depends upon the location and power of the source.

In Eq. (29) the reflection coefficients for the potentials are included. However, it can be said that the equation will not change if it is expressedin terms of the reflection coefficients which were determined as ratios between the vertical components of the velocities (or displacements) in corresponding reflected and incident waves.

Let us define these coefficients for each lower boundary by $Q^{(1)}_{11}$, $Q^{(1)}_{12}$, $Q^{(1)}_{21}$, and $Q^{(1)}_{22}$, so that

$$Q^{(1)}_{11}=\frac{w_{pp}}{w_p},\quad Q^{(1)}_{21}=\frac{w_{sp}}{w_s},\quad Q^{(1)}_{12}=\frac{w_{ps}}{w_p},\quad Q^{(1)}_{22}=\frac{w_{ss}}{w_s}, \qquad (30)$$

where $w_p$ and $w_s$ are the vertical displacements in the incident longitudinal and transverse waves respectively,* $w_{pp}$, $w_{ps}$ are the vertical displacements in the reflections of the PP, PS, etc., waves respectively.

It is not difficult to correlate the system of coefficients $V^{(1)}_{11}$, $V^{(1)}_{12}$ . . . with the system $Q^{(1)}_{11}$, $Q^{(1)}_{12}$, . . ., if we take into account the fact that the vertical component of the displacement is expressed in terms of the potentials by the following formula:

$$w=\partial\varphi/\partial z+\partial\psi/\partial x,$$

where the first term yields the displacement in the longitudinal waves and the second yields the displacement in the transverse waves.

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* Since a stationary oscillatory regime is being examined, it follows that the displacement should be taken to mean the amplitudes of the particle oscillations.

Without performing the actual mathematical operations, we write the final formulas:

$$\left.\begin{aligned} Q_{11}^{(1)} &= -V_{11}^{(1)}, & Q_{21}^{(1)} &= \frac{\alpha}{i\sigma} V_{21}^{(1)}, \\ Q_{12}^{(1)} &= -\frac{i\sigma}{\alpha} V_{12}^{(1)}, & Q_{22}^{(1)} &= V_{22}^{(1)}, \end{aligned}\right\} \tag{31}$$

where $\sigma = k \sin \vartheta$, $\alpha = ik \cos \vartheta$.

For the upper boundary the formulas are written in the same way, the only difference being that $\alpha$ is replaced by $-\alpha$.

Now it can easily be shown that Eq. (29) can be rewritten without change by means of the simple replacement of the coefficients V by the coefficients Q while retaining the corresponding indices.

In the case which is of greatest interest to us (the case of unattenuated normal waves – a case which involves a complete internal reflection of longitudinal and transverse waves at the boundary of the layer) Eq. (29) can be written in another form. Let us introduce the definitions

$$\left.\begin{aligned} g_1 &= -\frac{i\mu_1}{\mu}\alpha\beta_1, \quad h_1 = \frac{-i\mu_1}{\mu}\beta\alpha_1, \\ j_1 &= \frac{1}{\mu\varkappa^2\varkappa_1^2}[\sigma^2(\mu_1\xi_1 - \mu\xi)^2 - \alpha_1\beta_1(\mu\xi - 2\mu_1\sigma^2)^2], \\ k_1 &= -\frac{\alpha\beta}{\mu^2\varkappa^2\varkappa_1^2}[(\mu_1\xi_1 - 2\mu\sigma^2)^2 - 4\sigma^2\alpha_1\beta_1(\mu_1 - \mu)^2], \\ l_1 &= -\frac{4(\mu_1 - \mu)k\cos\vartheta}{\mu^2\varkappa^2\varkappa_1^2}[\sigma^2(\mu_1\xi_1 - 2\mu\sigma^2) + \alpha_1\beta_1(\mu\xi - 2\mu_1\sigma^2)], \end{aligned}\right\} \tag{32}$$

where

$$\left.\begin{aligned} \alpha &= ik\cos\vartheta, & \beta &= i\varkappa\cos\gamma, \\ \alpha_1 &= ik_1\cos\vartheta_1, & \beta_1 &= i\varkappa_1\cos\gamma_1, \\ \sigma &= k\sin\vartheta = k_1\sin\vartheta_1 = \varkappa\sin\gamma = \varkappa_1\sin\gamma_1, \end{aligned}\right\} \tag{33}$$

$k_1 = \dfrac{\omega}{c_1}$ and $\kappa_1 = \dfrac{\omega}{b_1}$ are respectively the wave numbers of the longitudinal and transverse waves in the medium which bounds the layer from below.

In terms of these definitions the reflection coefficients are written thus:

$$\left.\begin{aligned} V_{11}^{(1)} &= \frac{k_1 - j_1 + i(g_1 - h_1)}{k_1 + j_1 + i(g_1 + h_1)} = \Gamma_1 e^{-i(\bar{\varepsilon}_1 + \varepsilon_1)}, \\ V_{22}^{(1)} &= \frac{k_1 - j_1 - i(g_1 - h_1)}{k_1 + j_1 + i(g_1 + h_1)} = \Gamma_1 e^{-i(\varepsilon_1 - \bar{\varepsilon}_1)}, \end{aligned}\right\} \tag{34}$$

where

$$\left.\begin{aligned} \Gamma_1 &= \sqrt{\frac{(k_1 - j_1)^2 + (g_1 - h_1)^2}{(k_1 + j_1)^2 + (g_1 + h_1)^2}}, \\ \varepsilon_1 &= \operatorname{arc\,tan}\frac{g_1 + h_1}{k_1 + j_1}, \quad \bar{\varepsilon}_1 = \operatorname{arc\,tan}\frac{g_1 - h_1}{k_1 - j_1} \end{aligned}\right\} \tag{35}$$

and further

$$\left.\begin{aligned} V_{12}^{(1)} &= \Lambda_1 e^{-i\varepsilon_1}, \quad \Lambda_1 = \frac{\sigma l_1}{\sqrt{(k_1+j_1)^2+(g_1+h_1)^2}}, \\ V_{21}^{(1)} &= \Delta_1 e^{-i\varepsilon_1}, \quad \Delta_1 = \frac{\beta}{\alpha}\Lambda_1, \end{aligned}\right\} \tag{36}$$

$$V_{11}^{(1)}V_{22}^{(1)} - V_{12}^{(1)}V_{21}^{(1)} = e^{-2i\varepsilon_1} \tag{37}$$

Analogous relationships will also obtain for $V_{11}^{(2)}, \ldots$

Substituting these expressions for the reflection coefficients into the dispersion Eq. (29), we can write this equation in the form[*]

$$\cos[(k_z+\varkappa_z)h-\varepsilon_1-\varepsilon_2] = \Gamma_1\Gamma_2\cos[(k_z-\varkappa_z)h-\bar{\varepsilon}_1-\bar{\varepsilon}_2]-\Lambda_1\Delta_2. \tag{38}$$

From this it is also possible to obtain the dispersion equation for surface waves. For this purpose, as was the case in Section 1, the transition to the limit $|k_z+\kappa_z|h \to \infty, |\kappa_z-k_z|h \to \infty$ can be realized. Assuming the presence of some (even though small) attenuation of the waves in the layer, we obtain the relationship

$$\cos[(k_z-\varkappa_z)h-\bar{\varepsilon}_1-\bar{\varepsilon}_2]/\cos[(k_z+\varkappa_z)h-\varepsilon_1-\varepsilon_2] \to e^{2i\varkappa_z h} \to 0.$$

Thus under these conditions we obtain

$$1/\Gamma_1\Gamma_2 \to 0,$$

from Eq. (38), and this is satisfied when

$$1/\Gamma_1 \to 0 \text{ and } 1/\Gamma_2 \to 0. \tag{39}$$

Each of these relationships represents a dispersion equation for the surface waves on each of the boundaries.

## 3. Applicability to Any Selected Layer

Let us examine a medium which consists of any arbitrarily large number of plane-parallel liquid layers. The thickness of layer number $i$ shall be designated by $h_i$, the density of the medium in it shall be designated by $\rho_i$, and the velocity of sound in it shall be designated by $c_i$. The index $i$ shall apply to all integral values from $-\infty$ to $+\infty$.

If not one of these layers is separated from its neighbor by an absolutely reflecting boundary, then a unified system of normal waves is established in the examined medium, irrespective of the location of the radiator.[**] Let us demonstrate that the characteristics of these normal waves (the phase velocity and the attenuation) may be found from the dispersion Eq. (9) when it is applied to a layer of any arbitrary number $i$:

$$V_i^{(1)}V_i^{(2)}e^{2ik_i h_i\cos\vartheta_i} = 1, \tag{40}$$

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[*] As a result we arrive at the equation which was first obtained in [4].

[**] Only the excitation coefficients for various normal waves will depend upon the location of the radiator.

where $V_i^{(1)}$ and $V_i^{(2)}$ are the reflection coefficients for plane waves at the lower and upper boundaries of the layer i. The angles($\vartheta_i$) in the various layers are correlated with one another by the law of refraction

$$k_i \sin \vartheta_i = \text{const}.$$

Let us examine two arbitrarily selected neighboring layers to which we shall assign the indices $i = 0$ and $i = 1$ (Fig. 3).

The reflection coefficients $V_0^{(1)}$ and $V_0^{(2)}$ can be expressed in terms of $V_1^{(1)}$ and $V_1^{(2)}$ and in terms of the transparence coefficients of the boundary separating the two layers. In fact, $V_0^{(2)}$ is nothing less than the resultant coefficient of reflection from layer 1 and from the aggregate of all the subsequent layers $i = 2, 3, \ldots$.

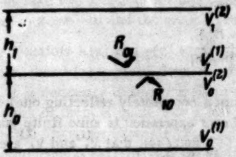

Fig. 3.

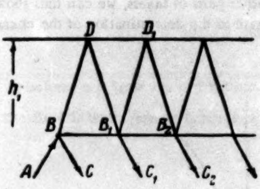

Fig. 4.

Let us symbolically depict a plane wave which is incident upon layer 1 from layer zero in the form of the ray AB in Fig. 4. This wave produces:

1) a directly reflected wave BC whose amplitude will be $R_{10}$, where $R_{10}$ is the reflection coefficient from the boundary separating the layers when a wave is incident upon it from the medium $i = 0$;

2) the wave $ABDB_1C_1$ which passes through the boundary twice and is reflected from the upper boundary of layer 1. The amplitude of this wave will be $V_1^{(2)}D_{10}D_{01}e^{2b_1}$, $b_1 = ik_1h_1 \cos \vartheta$ where $D_{01}$ and $D_{10}$ are the transparence coefficients of the boundary separating the layers when a wave is incident upon it from the directions of medium $i = 0$ and medium $i = 1$ respectively;

3) the wave $ABDB_1D_1B_2C_2$ with the amplitude $D_{10}D_{01}(V_1^{(2)})^2R_{01}e^{4b_1}$, etc.

Adding all these waves, we obtain the following expression for the amplitude of the resultant reflected wave (numerically, this amplitude is equal to the coefficient of reflection $V_0^{(2)}$):

$$V_0^{(2)} = R_{10} + V_1^{(2)} D_{10} D_{01} e^{2b_1} + D_{10} D_{01} (V_1^{(2)})^2 R_{01} e^{4b_1} + \cdots \tag{41}$$

If we assume the wave amplitude to mean the amplitude of acoustic pressure, then on the basis of the continuity condition for the pressure during the transition through the boundary we have the relationships

$$1 + R_{10} = D_{10}, \quad 1 + R_{01} = D_{01}.$$

In addition, we have $R_{10} = -R_{01}$ from the known expressions for the coefficient of reflection from the separation boundary between the two media.

Utilizing the formula for an infinite geometric progression in Eq. (42) we obtain:

$$V_0^{(2)} = R_{10} + \frac{V_1^{(2)} D_{10} D_{01} e^{2b_1}}{1 - V_1^{(2)} R_{01} e^{2b_1}} = \frac{R_{10} + V_1^{(2)} e^{2b_1}}{1 + R_{10} V_1^{(2)} e^{2b_1}} \tag{42}$$

By means of the same type of reasoning it is possible to obtain

$$V_1^{(1)} = \frac{R_{01} + V_0^{(1)} e^{2b_0}}{1 + R_{01} V_0^{(1)} e^{2b_0}}, \quad b_0 = ik_0 h_0 \cos \vartheta_0. \tag{43}$$

Further, from Eq. (42) we find that

$$V_1^{(2)} e^{2b_1} = \frac{V_0^{(2)} - R_{10}}{1 - V_0^{(2)} R_{10}}. \tag{44}$$

Making use of Formulas (43) and (44) we obtain

$$V_1^{(1)} V_1^{(2)} e^{2b_1} - 1 = \frac{(V_0^{(1)} V_0^{(2)} e^{2b_0} - 1)(1 - R_{01}^2)}{(1 - V_0^{(2)} R_{10})(1 + R_{01} V_0^{(1)} e^{2b_0})}. \tag{45}$$

Let us assume that $R_{01}^2 \neq 1$ (i. e., the boundary between layers is not a completely reflecting one), and that the expression in each of the parentheses in the denominator of the latter expression is some finite quantity which is different from zero or infinity; this means, in accordance with (43) and (44), that $V_1^{(1)}$ and $V_1^{(2)}$ are finite quantities. Then from the latter expression it follows that the difference $V_1^{(1)} V_1^{(2)} e^{2b_1} - 1$ becomes zero simultaneously with the difference $V_0^{(1)} V_0^{(2)} e^{2b_0} - 1$.

Successively repeating the line of reasoning given above for the other pairs of layers, we can thus show that all equations of the type exemplified by Eq. (40) are equivalent with regard to the determination of the characteristics of normal waves.

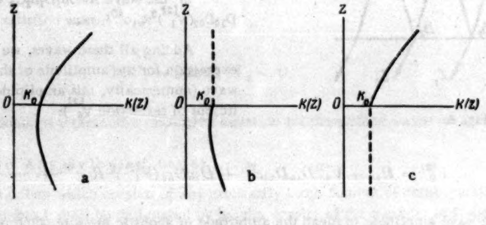

Fig. 5.

Our reasoning can be generalized to include the case when the parameters of the medium vary continuously, either within certain intervals of the values of z (the coordinate z is assumed to be directed normal to the boundaries of the layers) or for all values of z in the range of from $-\infty$ to $+\infty$. Under these conditions the dispersion equation is written in the form of Formula (5) and refers to a definite level (value of z) with respect to which the reflection coefficients $V_1$ and $V_2$ are determined. Allowing the thickness of the layers $h_i$ to go to zero in the reasoning given above, and allowing the number of layers to approach infinity, we can make the transition to the case of the continuous variation of the parameters; we can thus show that in order to determine the characteristics of normal waves we may choose any level.

However, it is also possible to demonstrate the same thing independently, without the above-mentioned transition in the limit. For this purpose let us examine the case when the parameters of the medium vary continuously everywhere. In that case the acoustic pressure is the solution of a second-order equation:

$$\Delta p + k^2(z)\, p = -\frac{1}{c^2} \operatorname{grad} \rho \operatorname{grad} p.$$

Let us take the solution of this equation (this solution is in the form $F(z)e^{i\sigma x}$, where $\sigma$ is arbitrary). Then we shall have the equation

$$\frac{d^2F}{dz^2} + [k^2(z) - \sigma^2]\, F = -\frac{1}{\rho}\frac{d\rho}{dz}\frac{dF}{dz}. \tag{46}$$

for F(z).

Let us take two solutions for this equation, $F_1(z)$ and $F_2(z)$, and let us assume that the first of them remains bounded when $z \to -\infty$ and the second remains bounded when $z \to \infty$.

The functions k(z) and $\rho(z)$ were selected to be arbitrary as long as they be continuous. Let us assume, for example, that k(z) is of the form shown in Fig. 5, a. Let us use the definition $k(0) = k_0$. In order to determine $V_1$ and $V_2$ it is necessary to imagine that the entire space is subdivided into two half-spaces $z > 0$ and $z < 0$, and that then each of them is made into a full space by a homogeneous medium (Figs. 5, b and 5, c) in which $k = k(0)$ and $\rho = \rho(0)$. Then, for example, $V_1$ is the coefficient of reflection of a plane wave which is incident upon the lower inhomogeneous half-space from the homogeneous medium.

In order to determine $V_1$ let us note that the field in the upper homogeneous medium can be written in the form

$$\varphi_0 = (e^{-i\alpha z} + V_1 e^{i\alpha z})\, e^{i\sigma x}, \tag{47}$$

where, as before, $\alpha = k_0 \cos\vartheta$, $\sigma = k_0 \sin\vartheta$, and the amplitude of the incident wave is assumed equal to unity.

The field in homogeneous half-space $z < 0$ is written as

$$\varphi_1 = W_1 F_1(z)\, e^{i\sigma x}, \tag{48}$$

where $W_1$ is some constant. The condition requiring the continuity of acoustic potential and of its derivative with respect to z when $z = 0$ yields two equations:

$$\left.\begin{aligned} 1 + V_1 &= W_1 F_1(0), \\ i\alpha(V_1 - 1) &= W_1 F_1'(0), \end{aligned}\right\} \tag{49}$$

From this we find

$$V_1 = \frac{i\alpha F_1(0) + F_1'(0)}{i\alpha F_1(0) - F_1'(0)} \tag{50}$$

In the same wave we find

$$V_2 = \frac{i\alpha F_2(0) - F_2'(0)}{i\alpha F_2(0) + F_2'(0)}. \tag{51}$$

Let us assume that the dispersion Eq. (5) corresponds to a level of $z = 0$. Then, substituting Expression (50) and (51) in Eq. (5), we obtain

$$V_1V_2 - 1 = \frac{2i\alpha\,[F_1'(0)F_2(0) - F_2'(0)F_1(0)]}{[i\alpha F_1(0) - F_1'(0)]\,[i\alpha F_2(0) + F_2'(0)]} = 0.$$

Let us assume that the expressions within the brackets in the denominator are finite quantities. Then in order for the dispersion equation to be satisfied it is necessary and sufficient that the Wronskian

$$w(0) = F_1'(0)F_2(0) - F_2'(0)F_1(0)$$

become equal to zero. However, since the value of the Wronskian does not depend upon the value of $z$, it follows that in order to determine the characteristics of normal waves it is possible to use any other level in place of the level $z = 0$; this is the very fact we sought to prove.

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# MOTION OF A GAS DUE TO THE EFFECT OF A BRIEF IMPULSE

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**The paper contains further refinements of the problem involved in integration of autosimulating equations in the problem concerning a shock upon a cold gas. The paper provides the results obtained from computations of the problem concerning the motion of a gas due to the effect of a brief finite impulse; these results illustrate the manner in which the motion acquires an autosimulating mode.**

In the paper by Zel'dovich [1] an investigation is made of plane unidimensional motion of an ideal gas with a specific heat ratio $\gamma$, a motion which arises when an instantaneous impulse is applied to the left-hand boundary of the gas; here the gas is assumed to be in a quiescent state at the initial instant $t = 0$, the density is assumed to be constant ($\rho = \rho_0$), the pressure is assumed to be $p = 0$. This motion proves to be an autosimulating motion. It is evident that at the initial instant $t = 0$ a shock wave arises which travels along the quiescent gas. Therefore this motion is described by the equations of hydrodynamics on the basis of the indicated initial data and the following boundary conditions: at the left – the conditions for a free boundary, at the right – the conditions for a shock wave. If we introduce the definitions: $\underline{t}$ – time, $\underline{r}$ – the Euler coordinate, $\underline{m}$ – the mass of the gas as measured from the boundary $r = 0$ at the instant $t = 0$, $\underline{u}$ – the velocity of a particle, $\rho$ – the density, $\underline{e}$ – the specific internal energy, $\underline{p}$ – the pressure, then the hydrodynamic equations will have the following form in Lagrangian coordinates:

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial m} = 0, \quad \frac{\partial \rho}{\partial t} + \rho^2 \frac{\partial u}{\partial m} = 0, \quad \frac{\partial \left(e + \frac{u^2}{2}\right)}{\partial t} + \frac{\partial (up)}{\partial m} = 0.$$

The mass M of the gas which is enclosed between the free boundary (i. e., the boundary with the condition $p = 0$) and the wave front is evidently a single-valued time function $M = M(t)$. If we designate the quantities in front of the shock wave front by the index "zero" and the quantities behind the shock wave front by the index "one" then the conditions on the shock wave can be written in the following form:

$$u_1^2 = p_1 \frac{\rho_1 - \rho_0}{\rho_1 \rho_0}; \quad \rho_1 = h\rho_0, \ \left(h = \frac{\gamma + 1}{\gamma - 1}\right); \quad \frac{dM}{dt} = \frac{p_1}{u_1}.$$

On the free boundary $m = 0$, and the pressure is $p = 0$.

Ia. D. Zel'dovich seeks a solution in the form

$$p = A\rho_0 M^{-n} f(\eta);$$
$$u = \sqrt{A} M^{-n/2} w(\eta);$$
$$\rho = \rho_0 q(\eta).$$

Here $\eta$ = m/M is the independent autosimulating variable, n is the autosimulating exponent (n > 0). A is a constant positive quantity which is determined by the magnitude of the instantaneous impulse. For the functions $f(\eta)$, $w(\eta)$, $q(\eta)$ we obtain a system of ordinary differential equations:

$$\begin{gathered} \sqrt{\frac{h}{h-1}}\,\eta\frac{dw}{d\eta}-\frac{df}{d\eta}=-\frac{n}{2}\sqrt{\frac{h}{h-1}}\,f; \\ \sqrt{\frac{h}{h-1}}\,\eta\frac{d\left(\frac{1}{q}\right)}{d\eta}+\frac{dw}{d\eta}=0,\ q=hf^{\frac{h-1}{h+1}}\eta^{\frac{h-1}{h+1}n} \end{gathered} \tag{1}$$

and the boundary conditions

$$\eta=1:\ f=1,\ q=1;\ w=\sqrt{\frac{h-1}{h}},\ \eta=0:\ f=0. \tag{2}$$

The line $\eta$ = 1 corresponds to the shock front, the line $\eta$ = 0 corresponds to the free boundary.

The autosimulating exponent n is determined from the condition which requires the existence of a solution of Equations (1) which satisfies the corresponding conditions. Ia. B. Zel'dovich has shown that 1 < n < 2. The method for determining the autosimulating exponent is developed in detail in the paper by Adamskii [2].

By means of changing variables:

$$f=\eta^{\frac{h+1}{h}-\frac{h-1}{2h}n}z^{\frac{h+1}{2h}},\quad w=\eta^{\frac{1}{h}-\frac{h-1}{2h}n}z^{\frac{h+1}{2h}}u.$$

Adamskii reduces the System (1) to one differential equation

$$\frac{du}{dz}=\frac{1}{z}\cdot\frac{n-\left(1+\frac{n}{2}\right)\sqrt{\frac{h}{h-1}}\,u+(h+1)\sqrt{\frac{h}{h-1}}\,uz-\frac{1}{2}\frac{h(h+1)}{h-1}nu^2}{\frac{h^2}{h-1}nuz+(2+n)\sqrt{\frac{h}{h-1}}-2\sqrt{\frac{h}{h-1}}\left(h+1-\frac{h-1}{2}n\right)}. \tag{3}$$

and the quadratures

$$\eta=\exp\left[\int_1^z\frac{h+1}{2h}\cdot\frac{\sqrt{\frac{h}{h-1}}\left(\frac{u}{z}+\frac{2h}{h+1}\frac{du}{dz}\right)-1}{\frac{h+1}{h}-\frac{h-1}{h}n-\frac{1}{h}\left(1+\frac{n}{2}\right)\sqrt{\frac{h}{h-1}}\,u}\,dz\right],$$

$$q=hf^{\frac{h-1}{h+1}}\eta^{\frac{h-1}{h+1}n}.$$

It is evident that the boundary conditions will be

$$u(1)=\sqrt{\frac{h-1}{h}},\ u(0)=-\infty.$$

Equation (3) has a singular saddle point with the coordinates

$$z=\frac{1}{h+1},\ u=-2\sqrt{\frac{h-1}{h}};$$

It follows from an analysis of the field of the integral curves for Eqs. (3) [2] that the autosimulating exponent n is determined from the condition which requires the existence of an integral curve satisfying Eq. (3) and passing through the point B $\left(z = 1,\ u = -\sqrt{\frac{h-1}{h}}\right)$ and the singular point C $\left(z = \frac{1}{h+1},\ u = -2\sqrt{\frac{h-1}{h}}\right)$. Since the determination of the autosimulating exponent n is performed by the probe method, it is desirable to have an "a priori" value of n which is accurate as possible.

Simple geometric concepts permitted us to appreciably narrow the interval of possible values of n, and in the case of a diatomic gas ($\gamma = 7/5$), which is examined in [2] it was possible to indicate an accurate value of the exponent n and a corresponding accurate solution of the System (1) which satisfied the boundary conditions.

In Eq. (2) we perform the change of variables:

$$z = \frac{1}{x},\quad u = u,$$

after which Eq. (2) will be of the form:

$$\frac{du}{dx} = \frac{\frac{1}{2}\frac{h(h+1)}{h-1}nu^2 + \left(1+\frac{n}{2}\right)\sqrt{\frac{h}{h-1}}ux - \sqrt{\frac{h}{h-1}}u - nx}{\frac{h^2}{h-1}nu + (2+n)\sqrt{\frac{h}{h-1}}x - 2\sqrt{\frac{h}{h-1}}\left[(h+1) - \frac{h-1}{2}n\right]} \cdot \frac{1}{x}. \tag{4}$$

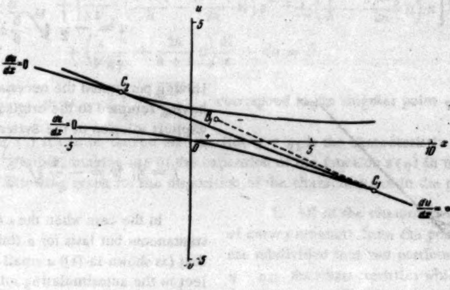

Fig. 1.

The point B will become the point $B_1$ $\left(x = 1,\ u = \sqrt{\frac{h-1}{h}}\right)$, the point C will become the point $C_1$ $\left(x = \frac{1}{h+1},\ u = -2\sqrt{\frac{h-1}{h}}\right)$. The problem consists of finding the exponent n for which there will be an integral curve for Eq. (3) passing through the points $B_1$ and $C_1$. Figure 1 shows a graph of the zero and infinity isoclines for Eq. (3); the signs of the derivatives in the corresponding regions are shown, and the points $B_1$ and $C_1$ are plotted. Since the unknown integral curve must pass through the points $B_1$ and $C_1$ and since the segment $B_1C_1$ intersects the isocline of infinities only at the singular point $C_1$, it follows that a point must exist on the segment $B_1C_1$ at which the field of the integral curves for Eq. (3) is tangent to the segment $B_1C_1$. The equation of the straight line which passes through the points $B_1$ and $C_1$ will be

$$u = -\frac{3}{h}\sqrt{\frac{h-1}{h}}x + \left(\frac{3}{h}+1\right)\sqrt{\frac{h-1}{h}}$$

and therefore the equation

$$-\frac{3}{h}\sqrt{\frac{h-1}{h}}=\frac{\frac{1}{2}\,\frac{h(h+1)}{h-1}uY^2+\left(1+\frac{h}{2}\right)Yx-\sqrt{\frac{h}{h-1}}\,Y-nx}{\frac{h^2}{h-1}nY+(2+n)\sqrt{\frac{h}{h-1}}\,x-2\sqrt{\frac{h}{h-1}}\left[h+1-\frac{h-1}{2}n\right]}\cdot\frac{1}{x} \tag{5}$$

$$Y=-\frac{3}{h}\sqrt{\frac{h-1}{h}}\,x+\left(\frac{3}{h}+1\right)\sqrt{\frac{h-1}{h}},$$

must have at least one root in the interval $1 \leq x \leq h+1$. Having performed the necessary mathematical operations, we obtain the following inequalities for the values of n:

$$\frac{2h+12}{h+12}\leqslant n\leqslant\frac{8h+12}{7h+3} \quad \text{for} \quad h\leqslant 6,$$

$$\frac{8h+12}{7h+3}\leqslant n\leqslant\frac{2h+12}{h+12} \quad \text{for} \quad h\geqslant 6.$$

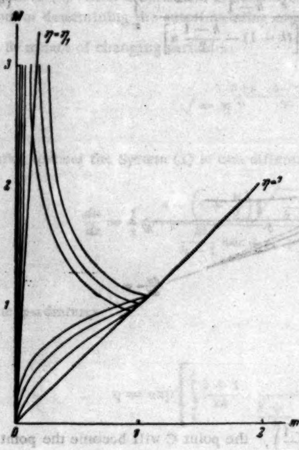

Fig. 2.

Therefore for a diatomic gas ($\gamma = 7/5$, $h = 6$) $n = 4/3$, and since when $n = 4/3$ and $h = 6$, Eq. (5) becomes an identity, it follows that in this case the unknown integral curve will be the straight line

$$u=-\frac{1}{2}\sqrt{\frac{5}{6}}\,x+\frac{3}{2}\sqrt{\frac{5}{6}}.$$

Having performed the necessary squaring operations and having returned to the original variables, we obtain an explicit solution of the System (1) ($h = 6$, $n = 4/3$):

$$f=\eta;\;\; w=-\frac{1}{2}\sqrt{\frac{5}{6}}\left(\eta^{-2/3}-3\right);\;\; q=6\eta^{8/3}.$$

In the case when the external impulse is not instantaneous but lasts for a finite time interval, it follows that (as shown in [1]) a small region which is not subject to the autosimulating solution exists in the vicinity of the free boundary; this region decreases with an increase in time. This is caused by the fact that the amount of energy enclosed between the free boundary and the wave front, as computed according to the autosimulating solution, is infinite. In fact, at a given instant of time (and therefore for a fixed value of M) the energy is

$$E=\int_{m}^{M}\left(e+\frac{u^2}{2}\right)dm=$$

$$=A^2M^{-n}\int_{z}^{1}\left(\frac{h-1}{2}\,\frac{f}{q}+\frac{w^2}{2}\right)dz=\frac{A^2M^{-n}}{n-1}\Bigg|_{z}^{1}-$$

$$-\sqrt{\frac{h-1}{h}}\,fw+z\left(\frac{h-1}{2}\,\frac{f}{q}+\frac{u^2}{2}\right).$$

Since when $\eta \to 0$ the asymptotic relationship

$$f \sim \eta^{\left(1+\frac{n}{2}\right)\frac{h+1}{h-1}-n};$$

$$w \sim \eta^{-n/2}; \quad q \sim \eta^{1+n/2},$$

obtains, it follows that when $\eta \to 0$, $E \sim M^{-1}m^{1-n}$.

From the formulas given above it is evident that with an increase in time (i. e., with an increase in the value of M) the region where the autosimulating regime is fulfilled (to a sufficient degree of accuracy) increases. In addition, we shall now show that the region which produces a divergence of the energy integral in the autosimulating solution does not affect the shock wave front. For this purpose we shall examine the disposition of the characteristic of this autosimulating solution in the plane m, M. The equations for the characteristics and for the relationships along the characteristics as expressed in Lagrangian coordinates, as we know, are of the form:

$$\frac{dm}{dt} = \pm c\rho, \quad \frac{dp}{\rho c} \pm du = 0. \tag{6}$$

In terms of the variables M, $\eta$, z and u these relationships can be rewritten in the following manner:

$$\begin{gathered} M\frac{d\eta}{dM} = \eta\left(\pm\sqrt{h+1}\,z^{1/2} - 1\right), \\ -\frac{n}{2}\left(\frac{2z^{-1/2}}{h\sqrt{\gamma}} \pm u\right)\frac{dM}{M} + \left[\frac{1}{h\sqrt{\gamma}}\left(\frac{h+1}{h} - \frac{h-1}{2h}n\right)z^{-1/2} \pm \left(\frac{1}{h} - \frac{h-1}{2h}n\right)u\right]\frac{d\eta}{\eta} + \\ +\left(\frac{z^{-1/2}}{\sqrt{h\gamma}} \pm \frac{2h}{h+1}u\right)\frac{dz}{z} \pm du = 0. \end{gathered} \tag{7}$$

Let the line $z = \frac{1}{h+1}$, $u = -2\sqrt{\frac{h-1}{h}}$ correspond to the singular point $\eta = \eta_1$. By the direct substitution of $\eta = \eta_1$ into Eq. (7) it can be proven that the line $\eta = \eta_1$ is the characteristic which corresponds to the plus sign in Formula (6). Further, making use of the expansion of the function $z(\eta)$ in the vicinity of the point $\eta = \eta_1$, we obtain the following graph for the disposition of the characteristics in the plane m, M (Fig. 2)

1. All of the characteristics of this same family of curves emanate from the point m = 0, M = 0 and are subdivided into two portions by the characteristic $\eta = \eta_1$: the characteristics which lie to the right of the line $\eta = \eta_1$ flow into the wave front line $\eta = 1$; each characteristic which lies to the left of the line $\eta = \eta_1$ does not intersect the wave front but asymptotically approaches a certain line m = const.

2. All of the characteristics belonging to the other family [i. e., those which correspond to the minus sign in Formula (6)] emanate from the line of the shock front $\eta = 1$, and each characteristic asymptotically approaches a certain line m = const. Thus the entire region which is enclosed between the free boundary m = 0 and the characteristic $\eta = \eta_1$ does not affect the wave front.

The autosimulating solution described above corresponds to the case where the impulse affecting the gas is instantaneous; however, if this impulse con-

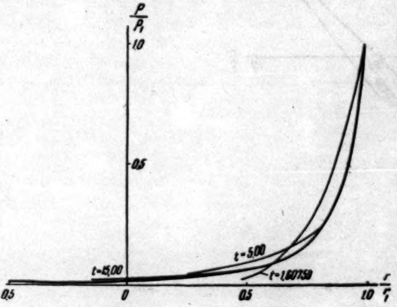

Fig. 3.

tinues for a certain finite time interval, the motion of the gas will not be rigorously autosimulating. Nevertheless, as the paper by Zel'dovich [1] asserts, it is to be expected that when a time interval expires which is large in comparison to the duration of the impulse the motion becomes approximately autosimulating, and thereafter the deviation of the regime of motion from an autosimulating regime will diminish without limit.

In order to clarify this fact the authors of this present paper undertook to compute such motion. We examined an ideal gas with a specific heat ratio of $\gamma = 7/5$ and an initial density of $\rho = 1$. The function $\pi(t)$, which describes the dependence of the external pressure upon time was determined in the following manner:

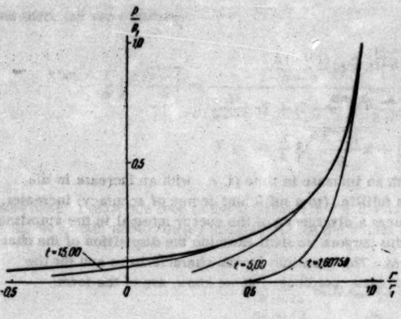

Fig. 4.

$$\pi(t) = 0,\ t < 0;\quad \pi(t) = 1,\ 0 \leqslant t < 1;\quad \pi(t) = 0,\ t \geqslant 1.$$

The computations were performed on a fast-response electronic computer according to the method of finite differences which are taken along the characteristics. The results clearly show the rapid rate at which the motion becomes autosimulating. We have provided graphs of the density $\rho$, the pressure $p$ and the velocity $u$ at certain instants of time as illustrations. The quantity $r/r_1$ is plotted along the axis of abscissas where $r$ is the current Euler coordinate, $r_1$ is the coordinate of the shock front at the corresponding instant in time; the quantities $\rho/\rho_1$, $p/p_1$ and $u/u_1$ are plotted along the axis of ordinates, where $\rho_1$, $p_1$, and $u_1$ again refer to the shock front.

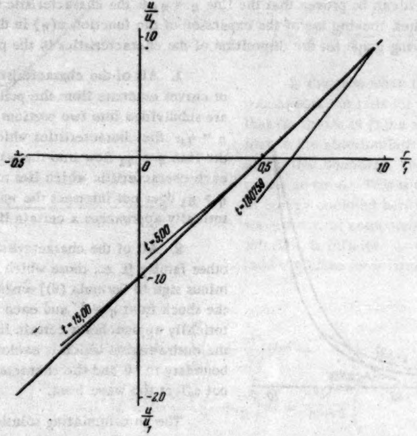

Fig. 5.

For the purposes of comparsion the curves which correspond to an accurate autosimulating solution have been plotted on the same coordinates.

These results permit us (for a specified shape of the external impulse) to compute the dimensionless constant B which appears in the expression for the pressure (see [1], p. 31)

$$p = \left(\frac{\tau \sqrt{P / \rho_0}}{X}\right)^n BPp.$$

This constant proves to be equal to 1.8224. It should be noted that in this particular case the autosimulating regime is established very rapidly – at the instant when the front boundary of the rarefaction wave (which is formed when the external pressure is removed) becomes a shock wave (this instant corresponds to the time t = = 1.60759) the nature of the distribution of the quantities p, ρ, and u is very close to being an autosimulating distribution (especially in the vicinity of the shock front) (Figs. 3, 4, 5).

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# THE THEORY GOVERNING THE SCATTERING OF SOUND BY A THIN ROD

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**The expression is derived for the acoustic pressure of the scattering field of a thin, infinitely long elastic rod of circular cross section; this expression takes into account both longitudinal and flexural vibrations of the rod. It is found that the vibrations of the rod can lead to a variation in the angular scattering characteristic. It is established that when sound is scattered by an elastic rod the phenomenon of space resonance may be observed when the phase velocity of the incident sound wave along the rod coincides with the velocity of flexural or longitudinal waves in a rod which is immersed in liquid. It is found that in the case of space resonance the scattering is substantially increased. As a particular case of the solution which is obtained, Rayleigh's formula is obtained for the scattering of a plane sound wave by a rod with a finite compressibility and density when the wave is incident perpendicular to the axis of the rod. It is noted that the internal losses in the material of the rod play a vital part in the scattering of the sound in the case of sufficiently thin rods.**

The problem involving the scattering of sound by a thin absolutely rigid and infinitely long cylinder (rod) is well known in acoustics [1]. However, the phenomena which are associated with flexural and longitudinal vibrations of a rod when scattering takes place have, as far as we know, not been examined up to this time.

When vibrations of the rod are taken into account the basis for the analysis consists of the approximate equation of motion which can be considered accurate in the case of thin rods. It is assumed that the flexural vibrations of the rod obey the wave equation for transverse vibrations in a thin rod, and that the longitudinal vibrations can be described by a wave equation which can be replaced by an equivalent equation for the transverse compression vibrations. The transverse compression vibrations of the rod are assumed to be its transverse symmetrical vibrations which are caused by the longitudinal vibrations and which are associated with the latter by the Poisson coefficient.

## The Scattering of Sound by a Thin Infinitely Long Rod in a Liquid

Let us choose a cylindrical system of coordinates $r$, $\varphi$, $z$ which is such that the axis of an infinitely long rod with a circular cross section having a radius $a$ coincides with the $z$ axis of the coordinate system. Let us assume that the rod is surrounded by an ideal liquid in which the propagation of vibrations can be described by the equation

$$\Delta p + k^2 p = 0, \qquad (1)$$

where $p$ is the sound pressure, $k = \frac{\omega}{c}$; $\omega$ is the angular frequency, $c$ is the velocity of sound in the liquid.

Let a plane monochromatic sound wave of a unit amplitude* (see figure) be incident at some angle $\theta$:

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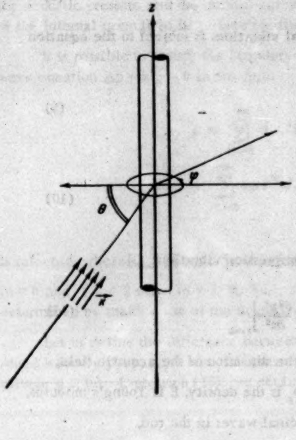
* The multiplier $\exp(-i\omega t)$ shall henceforth be omitted everywhere.

$$p_i = \exp[ik\cos\theta\cos\varphi r + ik\sin\theta z]\exp[-i\omega t]. \tag{2}$$

It is required to determine the acoustic field of the scattered wave.



Let us formulate the boundary conditions of the problem.

In the case of a rod which is sufficiently thin in comparison to a wavelength only two types of vibration can be usefully expressed out of all the possible types of vibration which arise in the rod due to the effect of the external field: flexural vibration, where the motion each element of therod reduces to a displacment in a direction perpendicular to the axis; symmetrical compressive vibration where deformation of an element does not depend upon the angle $\varphi$ around the axis of the rod.

For the purposes of simplification and ease in presenting the computations we shall make the initial approximate assumption that no other types of vibration except the two indicated ones can exist in the rod.

We shall assume that the normal rate of displacement of the surface of the rod will be of the following form in the general case:

$$v\cos\varphi + v_1, \tag{3}$$

where $\underline{v}$ and $v_1$ do not depend upon $\varphi$.

The first term is associated with the flexural vibration, and the second term is associated with the symmetrical vibration. At the boundary between the rod and the liquid, the normal components of the surface-displacement velocities of the rod must also be equal to

$$(v\cos\varphi + v_1)\,i\omega\rho = \left(\frac{\partial p}{\partial r}\right)_{r=a} \tag{4}$$

in the liquid. Here $\rho$ is the density of the liquid.

At infinity the quenchability condition must be fulfilled:

$$\lim_{r\to\infty}(p - p_i)\exp[-ik\sin\theta z] = 0 \quad \text{for} \quad \operatorname{Im} k > 0. \tag{5}$$

Conditions (4) and (5) would be sufficient for solving the problem which has been posed if the values of the surface-displacement velocities of the rod $\underline{v}$ and $v_1$ were known. In order to determine them let us make use of the equations of motion for the rod.

We shall represent the acoustic field in the liquid as a superposition of two fields – a symmetrical field $p''$ which is associated only with the symmetrical compression vibrations of the rod, and an unsymmetrical field $p'$ which is associated only with the unsymmetrical (flexural) vibrations of the rod*

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* Generally speaking, the unsymmetrical portion of the field is associated with all forms of vibration of the rod which are of the type $v\cos m\varphi$; however, we assume that all vibrations of the form $v\cos m\varphi$ are absent for $m = 2, 3, 4, \ldots$ (i. e., they are weakly expressed).

$$p = p' + p''. \tag{6}$$

In accordance with what has been said above,

$$p' = p'(r,\ \varphi)\exp[ik\sin\theta z], \tag{7}$$

$$p'' = p''(r,\ 0)\exp[ik\sin\theta z]. \tag{8}$$

The velocity of the normal surface displacement of the rod for flexural vibrations is subject to the equation of motion

$$g\frac{\partial^4 v}{\partial z^4} - \omega^2 m_s = v = (-i\omega F)_{r=a} \tag{9}$$

where

$$F = -a\int_0^{2\pi}(p)_{r=a}\cos\varphi\, d\varphi \tag{10}$$

and for symmetrical vibrations it is subject to the equation for transverse compression vibrations [3]

$$c_1^2\frac{\partial^2 v_1}{\partial z^2} + \omega^2 v_1 = \frac{i\omega^2 a(1-\sigma)}{E}\left[\left(1 - \frac{2\sigma^2}{1-\sigma}\right)p'' + \frac{c_1^2}{\omega^2}\frac{\partial^2 p''}{\partial z^2}\right]_{r=a}. \tag{11}$$

In Eqs. (9) and (11) F is the force acting upon a unit length of the rod from the direction of the acoustic field, $m_s = \pi a^2\rho_s$ is the mass of the rod per unit length, $g = \frac{E\pi a^4}{4}$ is the rigidity, $\rho_s$ is the density, E is Young's modulus, $\sigma$ is the Poisson coefficient of the rod, $c_1 = \sqrt{\frac{E}{\rho_s}}$ is the velocity of longitudinal waves in the rod.

We have the following expressions for the surface-displacement velocities of the rod:

$$v = v_0\exp[ik\sin\theta z], \tag{12}$$

$$v_1 = v_{01}\exp[ik\sin\theta z]. \tag{13}$$

Having made use of Eqs. (9-13), we can obtain the following expressions for the normal displacement velocities:

$$v = \left(\frac{F}{Z_1}\right)_{r=a}, \tag{14}$$

$$v_1 = -\left(\frac{p''}{Z_0}\right)_{r=a}, \tag{15}$$

where we have used the definitions

$$Z_1 = -i\omega m_s\left[1 - \left(\frac{c_u}{c}\sin\theta\right)^4\right], \tag{16}$$

$$Z_0 = i\frac{E}{\omega a}\frac{\left[1 - \left(\frac{c_1}{c}\sin\theta\right)^2\right]}{\left[1 - \sigma - 2\sigma^2 - (1-\sigma)\left(\frac{c_1}{c}\sin\theta\right)^2\right]}. \tag{17}$$

$c_u = \left(\omega^2\frac{g}{m_s}\right)^{1/4}$ — is the velocity of free flexural waves in the rod in a vacuum.

Equations (14) and (15) indicate the fact that the expressions for $Z_1$ and $Z_0$ can be called the impedances of the rod; here $Z_1$ is the impedance of the rod for flexural vibrations and represents the ratio between the force acting upon a unit length of the rod from the direction of the acoustic field and the normal component of the surface-displacement of the rod in the direction of the force (in the case of flexural vibrations); $Z_0$ is the impedance of the rod for transverse compression vibrations and represents the ratio between the symmetrical portion of the acoustic pressure and the normal component of the surface-displacement velocities of the rod in the direction of the internal normal to its surface (in the case of symmetrical vibrations of the rod).

It is possible to satisfy the Boundary Condition (4) and the Quenchability Condition (5) if a solution of the wave equation $\Delta p + k^2 p = 0$ in the form

$$p = \sum_{m=0}^{\infty} A_m H_m^{(1)} (k \cos \theta r) \cos m \varphi \exp [ik \sin \theta z] + \\ + \sum_{m=0}^{\infty} \varepsilon_m i^m J_m (k \cos \theta r) \cos m \varphi \exp [ik \sin \theta z] , \tag{18}$$

is selected, where $H_m^{(1)} (k \cos \theta r)$, $J_m (k \cos \theta r)$ -- are Hankel and Bessel function of the first kind, $\epsilon_m = 1$ when $m = 0$ and $\epsilon_m = 2$ when $m = 1, 2, 3, \ldots$ The unknown amplitudes of the partial cylindrical waves $A_m$ can be determined by making use of the Boundary Conditions (4).

Let us define the difference between the total acoustic pressure in the liquid and the pressure in the incident sound wave by $p_s$; $p_s$ represents the pressure in the field of the scattered wave. Substituting the values for the amplitude $A_m$ into Expression (18), we obtain the following equation for the scattered wave:

$$p_s = - \frac{J_0'(\alpha)}{H_0'^{(1)}(\alpha)} \frac{i\rho c \dfrac{J_0(\alpha)}{J_0^1(\alpha)} + Z_0 \cos \theta}{i\rho c \dfrac{H_0^1(\alpha)}{H_0'^{(1)}(\alpha)} + Z_0 \cos \theta} H_0^{(1)} (k \cos \theta r) \exp [ik \sin \theta z] - \\ - 2i \frac{J_1'(\alpha)}{H'^{(1)}(\alpha)} \frac{i\rho c \dfrac{J_1(\alpha)}{J_1'(\alpha)} + Z_1 \cos \theta}{i\rho c \dfrac{H_1^{(1)}(\alpha)}{H_1'^{(1)}(\alpha)} + Z_1 \cos \theta} H_1^{(1)} (k \cos \theta r) \cos \varphi \exp [ik \sin \theta z] + \Sigma', \tag{19}$$

where

$$J_m'(\alpha) = \left[ \frac{\partial J_m (k \cos \theta r)}{\partial (k \cos \theta r)} \right]_{r=a} ; \quad H_m'^{(1)} (\alpha) = \left[ \frac{\partial H_m^{(1)} (k \cos \theta r)}{\partial (k \cos \theta r)} \right]_{r=a} \alpha \equiv k \cos \theta a. \tag{20}$$

In Eq. (19) the first term describes the scattered wave which is associated with the symmetrical vibrations; the second term is associated with flexural (unsymmetrical)vibrations of the rod.

$\Sigma'$ represents the sum of the partial cylindrical waves for which the rod is an absolutely rigid obstacle. In actual fact, as we have indicated above, the rod can perform symmetrical vibrations for which the displacement does not depend upon the coordinate $\varphi$; it can also perform unsymmetrical vibrations for which the displacements are proportional to $\cos \varphi$. All of the surface displacements of the form $v \cos m\varphi$ ( $m = 2, 3, \ldots$) are so weakly expressed that for them the rod can be considered incompressible.

Thus $\Sigma'$ represents a portion of the known solution which corresponds to the case of the scattering of sound by a thin absolutely rigid and immobile rod which is infinitely long.

Taking into account the fact that $\Sigma' \rightarrow 0$ in the case of thin rods (since the values of the Hankel and Bessel functions are practically equal to zero if the number of the function is greater than its argument), we find that the scattering field is described by cylindrical functions of zero and first order only; these functions are associated with the symmetrical and flexural vibrations of the rod.

Let us analyze Expression (19).

## Space Resonance of Transverse Compression Vibrations of the Rod

Let us examine how the amplitude of the symmetrical portion of the scattered wave varies as a function of the angle of incidence of the plane sound wave upon the rod. Since the rod is thin ($\alpha \ll 1$), the cylindrical functions $J_0(\alpha)$, $H_0^{(1)}(\alpha)$ can be replaced by their values in the vicinity of zero. *

$$J_0(\alpha) \approx 1 - \frac{\alpha^2}{4}; \; H_0^{(1)}(\alpha) \approx \frac{2i}{\pi}\left[\ln \alpha - \frac{1}{2} i\pi\right]; \; J_0'(\alpha) \approx -\frac{\alpha}{2}; \; H_0'^{(1)}(\alpha) \approx \frac{2i}{\pi\alpha}. \tag{21}$$

By examining the field at great distances $k \cos \theta r \gg 1$, it is possible to make use of the asymptotic expression for the Hankel function:

$$H_m^{(1)}(k \cos \theta r) \approx \left(\frac{2}{\pi k \cos \theta r}\right)^{1/2} (-i)^m \exp\left[i\left(k \cos \theta r - \frac{\pi}{4}\right)\right]. \tag{22}$$

Having made these substitutions and having substituted the value of the impedance $Z_0$ from Eq. (17), we obtain a final expression for the symmetrically scattered field at great distances on the basis of Eq. (19):

$$p_s' = -\frac{\pi^2 a^2 (\cos \theta)^{3/2}}{\lambda^{3/2} r^{1/2}} \times$$

$$\times \frac{\dfrac{E}{2} \dfrac{\left[1 - \left(\frac{c_1}{c} \sin \theta\right)^2\right] \cos \theta}{\left[1 - \sigma - 2\sigma^2 - (1-\sigma)\left(\frac{c_1}{c} \sin \theta\right)^2\right]} - \dfrac{\rho c^2}{\cos \theta}}{\dfrac{E}{2} \dfrac{\left[1 - \left(\frac{c_1}{c} \sin \theta\right)^2\right] \cos \theta}{\left[1 - \sigma - 2\sigma^2 - (1-\sigma)\left(\frac{c_1}{c} \sin \theta\right)^2\right]} - \dfrac{1}{4}\omega^2 \rho a^2 i\pi \cos \theta - \omega^2 \dfrac{1}{2} \rho a^2 \cos \theta \ln \dfrac{1}{k \cos \theta a}} \times$$

$$\times \exp\left[ik \cos \theta r + ik \sin \theta z + \frac{\pi}{4}\right], \tag{23}$$

where $\lambda$ is the wavelengh in the liquid.

In Expressions (23) the typically "resonant" denominator attracts attention. The term $\frac{1}{2}\rho a^2 \ln \frac{1}{k \cos \theta a}$ represents (to an accuracy of a constant multiplier) a special kind of coupled mass, and the expression

$$\frac{E}{2} \frac{\left[1 - \left(\frac{c_1}{c} \sin \theta\right)^2\right] \cos \theta}{\left[1 - \sigma - 2\sigma^2 - (1-\sigma)\left(\frac{c_1}{c} \sin \theta\right)^2\right]}$$

represents the elasticity of the rod; the magnitude of this elasticity depends upon the angle of incidence of the plane sound wave upon the rod. The expression $1/4\omega^2 \rho a^2 \cos \theta\pi$ represents (to an accuracy of a constant multiplier) the radiation impedance of a cylindrical source of the symmetrical type whose dimensions are small in comparison to the wavelength of the sound wave. If a plane sound wave is incident upon the rod at such an angle that the condition

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* See, for example, [4], p. 352.

$$\frac{E}{2}\frac{\left[1-\left(\frac{c_1}{c}\sin\theta\right)^2\right]}{\left[1-\sigma-2\sigma^2-(1-\sigma)\left(\frac{c_1}{c}\sin\theta\right)^2\right]}-\omega^2\frac{1}{2}\rho a^2\ln\frac{1}{k\cos\theta a}=0, \tag{24}$$

is fulfilled, then we observe the phenomenon of resonance of the transverse compression vibrations of the rod, and the amplitude of the symmetrically scattered wave is determined by the expression*

$$\dot{p}_{0s}\simeq i\left(\frac{\lambda}{r\cos\theta}\right)^{1/2}\frac{1}{\pi}\exp\left[i\left(k\cos\theta r+k\sin\theta z+\frac{\pi}{4}\right)\right]. \tag{25}$$

By comparing the amplitudes of the symmetrical scattered wave at resonance and for the case of scattering by an absolutely rigid rod when a plane sound wave is perpendicularly incident upon the rod, we obtain

$$\left|\frac{p''_{0s}}{p_s}\right|\approx\frac{4}{\pi k^2a^2\cos^{1/2}\theta}. \tag{26}$$

From this expression it follows that in the case of a sufficiently thin rod the amplitude of the scattered wave at resonance may be substantially greater than in the case of scattering by an absolutely rigid rod when the sound wave is perpendicularly incident. Another no less interesting case is the case when the elasticity of the rod is equal to the elasticity of the liquid, i. e., when

$$\frac{E}{2}\frac{\left[1-\left(\frac{c_1}{c}\sin\theta\right)^2\right]\cos\theta}{\left[1-\sigma-2\sigma^2-(1-\sigma)\left(\frac{c_1}{c}\sin\theta\right)^2\right]}-\frac{\rho c^2}{\cos\theta}=0. \tag{27}$$

In the case being examined the symmetrical scattering field is equal to zero.

Finally when the condition

$$\frac{c_1}{c}\sin\theta=1 \tag{28}$$

obtains, the elasticity of the rod is equal to zero, and the rod behaves as an absolutely soft obstacle with regard to the symmetrical portion of the acoustic field.

As can be seen from Eq. (28), this is observed at an angle of incidence $\theta$ which is such that the phase velocity of the incident wave along the rod in the liquid is equal to the phase velocity of longitudinal waves in the rod. In this regard it is important to know that the phenomenon of resonance occurs for the condition which obtains when the phase velocity of the incident wave along the rod in the liquid becomes equal to the phase velocity of the longitudinal waves in a rod which is immersed in liquid.** Resonance in this case can be called space resonance [5].

## Space Resonance for Flexural Vibrations of the Rod

Let us examine the nature of the amplitude variations of the unsymmetrical scattered wave which is associated with flexural vibrations of the rod; let us examine these amplitude variations as a function of the angle

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* See supplement.

** The velocity of longitudinal waves in a rod immersed in liquid is less than the velocity of longitudinal waves in a rod in a vacuum; this is true because of the effect of the liquid upon the vibrations of the rod [3].

**of incidence of a plane sound wave upon the rod. Replacing, as we did above, the cyclindrical functions $J_1(\alpha)$, $H_1^{(1)}(\alpha)$ by their values in the vicinity of zero:**

$$J_1(\alpha) \approx \frac{\alpha}{2}, \quad H_1^{(1)}(\alpha) \approx -\frac{2i}{\pi\alpha} + \frac{\alpha}{2}; \quad J_1'(\alpha) \approx \frac{1}{2}, \quad H_1'^{(1)}(\alpha) \approx \frac{2i}{\pi\alpha^2}; \tag{29}$$

**as well as substituting the value of the impedance $Z_1$ from Eq. (16) and examining the field at great distances (22), we obtain from Eq. (19) a final expression for the unsymmetrical scattered field:**

$$p_s' = \frac{2\pi^2 a^2 (\cos\theta)^{3/2}}{\lambda^{3/2} r^{1/2}} \frac{\rho_s\left[1-\left(\frac{c_u}{c}\sin\theta\right)^4\right]-\rho}{\rho_s\left[1-\left(\frac{c_u}{c}\sin\theta\right)^4\right]+\rho-\frac{1}{4}\, i\rho\,\frac{\pi\omega^2 a^2}{c^2}\cos^2\theta} \times \\ \times \cos\varphi \exp\left[i\left(k\cos\theta\, r + k\sin\theta z + \frac{\pi}{4}\right)\right]. \tag{30}$$

**The expression $\rho_s\left[1-\left(\frac{c_u}{c}\sin\theta\right)^4\right]$ characterizes the "apprent density" of the rod. The term $\frac{1}{4}\rho\frac{\pi\omega^2 a^2}{c^2}\cos^2\theta$ represents (to an accuracy of a constant multiplier) the radiation impedance of an oscillating cylindrical source whose dimensions are smaller than a wavelength.**

**From Expression (30) it is evident that when the phase velocity of the incident sound wave along the rod coincides with the velocity of flexural waves in a rod which is located in a vacuum, i. e., when**

$$\frac{c_u}{c}\sin\theta = 1, \tag{31}$$

**then the apparent density of the rod is equal to zero and the rod practically behaves as an absolutely soft obstacle with regard to the antisymmetrical portion of the acoustic field.**

**When the apparent density of the rod is equal to the density of the liquid, the antisymmetrical portion of the scattered field is not present. The scattering of the sound is caused solely by the symmetrical vibrations of the rod. The rod flexes freely due to the effect of the acoustic field.**

**Finally, if the condition**

$$\rho_s\left[1-\left(\frac{c_u}{c}\sin\theta\right)^4\right]+\rho = 0, \tag{32}$$

**is fulfilled we observe space resonance of the flexural vibrations of the rod.**

**Taking Condition (32) into account, it follows from Eq. (30) that the amplitude of the antisymmetrical scattered wave at resonance is determined by the expression**

$$p_{0s}' \simeq +\, 4i\left(\frac{\lambda}{r\cos\theta}\right)^{1/2}\frac{1}{\pi}\cos\varphi\exp\left[i\left(k\cos\theta\, r + k\sin\theta\, z + \frac{\pi}{4}\right)\right] \tag{33}$$

**As in the case of longitudinal vibrations, the scattering is substantially increased when resonance of the flexural vibrations of the rod occurs.**

**It is important to underline the fact that space resonance of the flexural vibrations occurs in the case when the impedance of the rod is of an elastic nature and proves to be equal in magnitude to the inertial impedance of the liquid in cases where this impedance is associated with the coupled mass (in other words when the coupled mass is compensated by the elasticity of the rod).**

The velocity of flexural waves in the rod, in contrast to the velocity of longitudinal waves, depends upon the thickness of the rod and the frequency of the vibrations.

In the case of sufficiently thin rods the velocity of the flexural waves will be less than the velocity of sound in the liquid, and no space resonance of the flexural vibrations will be observed.

The criterion for the absence of space resonance of flexural vibrations is thus the relationship

$$c_u = \sqrt{\omega}\left(\frac{g}{m_s}\right)^{1/4} < c \tag{34}$$

The latter relationship can be rewritten in the following form:

$$fa < \frac{1}{\pi}\sqrt{\frac{c\rho_s}{E}} \text{ cps cm.} \tag{35}$$

For steel and water, for example, Expression (35) is of the form

$$fa < 2 \cdot 10^4 \text{ kc cm}$$

## Perpendicular Incidence of a Plane Sound Wave Upon the Rod. Rayleigh's Formula

Assuming that the angle of incidence is $\theta = 0$ in Relationships (23) and (30), taking into account the fact that

$$\frac{1}{2}\omega^2 \rho a^2 \ln\frac{1}{ka} \ll \frac{E}{2(1-\sigma-2\sigma^2)}$$

in the case of sufficiently thin rods, and also using the definitions

$$\frac{E}{2(1-\sigma-2\sigma^2)} \equiv m\,,\ \rho_c{}^2 \equiv m,$$

we obtain the Rayleigh formula for the scattering of a plane sound wave by a thin rod which has a finite compressibility and density when the wave is incident in the direction perpendicular to the axis of the rod* [1, § 343].

$$\operatorname{Re} p_{s|\theta=0} \approx -\frac{2\pi^2 a^2}{\lambda^{3/2} r^{1/2}}\left[\frac{m'-m}{2m'} - \frac{\rho_s-\rho}{\rho_s+\rho}\cos\varphi\right]\cos\left[\frac{2\pi}{\lambda}\left(ct-r-\frac{\lambda}{8}\right)\right]. \tag{36}$$

Assuming that the rod has an infinitely large density and an infinitely large elasticity ($\rho_s \to \infty$, $m' \to \infty$), we obtain the well-known Rayleigh formula for the pressure of the scattered wave in the case of an absolutely rigid and immobile rod:

$$\operatorname{Re} p_{s|\theta=0} \approx -\frac{2\pi^2 a^2}{\lambda^{3/2} r^{1/2}}\left[\frac{1}{2} - \cos\varphi\right]\cos\left[\frac{2\pi}{\lambda}\left(ct-r-\frac{\lambda}{8}\right)\right]. \tag{37}$$

Thus, in examining the scattering of sound by a thin rod in liquid, it is impermissible to fail to take into account the flexural and longitudinal vibrations of the rod.

In a number of cases the vibrations of the rod substantially affect the polar scattering characteristic and the magnitude of the scattered energy.

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* Expression (37) differs from the Rayleigh formula in terms of the sign in front of the term describing the antisymmetrical portion of the scattered wave. This is due to the fact that Rayleigh examined the scattering of a plane wave which was propagated in the direction $\varphi = \pi$ and not in the direction $\varphi = 0$ as in our case.

In conclusion I express my deep appreciation to L. M. Brekhovskikh and G. D. Maliuzhinets for their discussion of the problems discussed above and for a number of valuable remarks which they made.

## SUPPLEMENT

### The Part Played by the Internal Losses in the Material of the Rod

From Eq. (26) for the ratio between the amplitude of the scattered symmetrical wave at resonance and the amplitude of the scattered wave in the case of a rigid and immobile rod (when the wave is incident in a direction perpendicular to the axis of the rod) we have a paradoxical result: this ratio becomes infinite in the case of an infinitely thin rod: *

$$\left|\frac{\overset{*}{P}_{s0}}{P_s}\right| \approx \frac{4}{\pi k^2 a^2 \cos^{1/2}\theta\pi} \xrightarrow[ka\to 0]{} \infty \tag{38}$$

The paradoxical nature of this result is due to the idealization of the problem. We have not taken into account the internal losses in the material of the rod. The part played by these losses can be evaluated if we assume that Young's modulus of the material of the rod represents a complex quantity and can be described by the expression $E^* = E(1-i\eta)$, where $\eta$ is the loss coefficient. The impedances of the rod will now contain a real portion which characterizes the resistance of the internal losses.

For longitudinal vibrations of the rod it follows that $R_{0\,\mathrm{int}} \approx \frac{E\eta}{2\omega a\sigma^2} \xrightarrow[a\to 0]{} \infty$ Therefore when the thickness of the rod goes to zero, it follows that $R_{0\,\mathrm{int}} \to \infty$ (i. e., the resistance of the internal losses becomes infinitely large). The denominator of Expression (38) will now contain a term which corresponds not only to the radiation impedance but also to the resistance of the internal losses; this means that the value of the indicated Ratio (38) is equal to zero when $ka \to 0$. The paradoxical nature of the result disappears.

Thus in the case of very thin rods the losses in the material of the rod play a vital part in the scattering of the sound.

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* In the case of flexural vibrations this does not occur, since in very thin rods no resonance of the flexural vibrations is observed.

# CERTAIN CONSEQUENCES WHICH STEM FROM THE THEORY OF THE DIFFRACTION OF SOUND UPON A FLEXIBLE SPHERE

**S. N. Rzhevkin**

**The paper gives an additional analysis of the solution obtained by Rayleigh for the problem involving the diffraction of a plane wave upon a flexible sphere which is small in comparison with the wavelength. It is shown that the amplitude of the pulsating vibration of the surface of the sphere is determined by the complex impedance that depends upon the coupled mass, the internal elasticity and the radiation impedance of the sphere. The resonant frequency of the flexible sphere is found by setting the reactance equal to zero. It is shown that a resonating gaseous sphere in a liquid sucks in an amount of energy equal to $\lambda^2/\pi$ from the wave front.**

**The expression is found for the amplitude of the oscillating vibrations of of the sphere as a function of the relationship between the densities of the internal ($\rho_1$) and external ($\rho$) media; the degree to which the viscosity affects the vibrations is demonstrated.**

Rayleigh[1] solved the problem of the diffraction of a plane sound wave upon a sphere (liquid or gaseous) whose density and modulus of elasticity ($\rho_1$, $\varkappa_1$) differ from the density and elasticity ($\rho$, $\varkappa$) of the surrounding medium. A solution of the problem is given in the form of an expansion in terms of spherical functions of various orders m = 0, 1, 2, . . . The scattered (diffracted) pressure wave can be represented in the form

$$p_s = \sum_{m=0}^{\infty} a_m P_m(\vartheta)\, h_m^{(2)}(kr)\, e^{j\omega t}, \qquad (1)$$

where $P_m(\vartheta)$ is a Legendre polynomial, $\vartheta$ is the polar angle, i. e., the angle between a direction (from the center of the sphere) toward a specified point (lying at a distance $\underline{r}$ from the center) and a direction toward an infinitely remote source which lies on the $\underline{x}$ axis and which creates a plane wave with a sound-pressure amplitude $p_0$ and a velocity-amplitude $q_0 = \frac{p_0}{\rho c}$; in the vicinity of the sphere; $h_m^{(2)}(z)$ is a Hankel spherical function of the second kind and of the $\underline{m}$th order z = kr.

The first three coefficients of the Series (1) can be represented by the following expressions with an accuracy which is sufficient for the subsequent analysis, provided that the condition $z_0 = kr_0 \ll 1$ (where $r_0$ is the radius of the sphere) is fulfilled:

$$a_0 \simeq jp_0 \frac{z_0^3}{3} \frac{1-\frac{\varkappa}{\varkappa_1}}{1-\frac{\varkappa}{\varkappa_1}\frac{z_0^2}{3}e^{-jz_0}};$$

$$a_1 \simeq p_0 z_0^3 \frac{\rho_1-\rho}{2\rho_1+\rho};\quad a_2 \simeq jp_0 \frac{2}{9} z_0^5 \frac{\frac{\rho_1}{\rho}-1}{2+3\frac{\varkappa_1}{\varkappa}}. \qquad (2)$$

Thus terms of the order 0 and 1 play the main part for small values of $z_0$; these terms include the multiplier $z_0^3$, whereas the term of order 2 contains the multiplier $z_0^5$.

Let us make use of Expressions (2) for $a_0$ (taking into account the fact that when $z_0 \ll 1$, $e^{-jz_0} \simeq 1 - jz_0$), in order to determine the radial velocity component on the surface of the sphere. The computation yields

$$q_r(r_0) \simeq q_{ir}(r_0) + q_{sr}(r_0) \simeq -\frac{p_0}{\rho c}\left[\gamma \frac{z_0}{3} + \frac{1 - \frac{\varkappa_1}{\varkappa}}{z_0^2 + j\left(z_0 - \frac{3}{z_0}\frac{\varkappa_1}{\varkappa}\right)}\right]. \tag{3}$$

The first term in this expression yields the radial velocity component of the incident plane wave; the second is due to the scattered wave.

With the exception of the case when $\kappa_1 \gg \kappa$ for $z_0 \ll 1$, the resultant radial velocity is determined solely by the second term in Eq. (3):

$$q_r(r_0) \simeq -p_0 \frac{1 - \frac{\varkappa_1}{\varkappa}}{\rho c\left[z_0^2 + j\left(z_0 - \frac{3}{z_0}\frac{\varkappa_1}{\varkappa}\right)\right]} = -p_0 \frac{1 - \frac{\varkappa_1}{\varkappa}}{Z_1}. \tag{3a}$$

The expression in the denominator,

$$Z_1 = \rho c\left[z_0^2 + j\left(z_0 - \frac{3}{z_0}\frac{\varkappa_1}{\varkappa}\right)\right] \tag{4}$$

represents the specific impedance of a spherical radiator; in this expression the first term is due to the radiation impedance of a pulsating sphere (when $z_0 \ll 1$); the second term represents the inertial impedance (this impedance is due to the coupled mass), and the third, as can easily be shown, is due to the elasticity of the volume of the sphere (per unit area).

If $\kappa_1 \gg \kappa$, then in accordance with Eq. (3a) we obtain $q_r(r_0) = 0$ (i. e., there is no deformation). If the imaginary portion of $Z_1$ is equal to zero, then resonance occurs at the frequency

$$z_{res} = \sqrt{\frac{3\varkappa_1}{\varkappa}}. \tag{5}$$

For a gas bubble in sea water we obtain $z_{res} \approx \frac{1}{68} = 1.44 \cdot 10^{-2}$ at atmospheric pressure, and the resonant frequency will be equal to $f_{res} \approx \frac{340}{r_0} = \frac{680}{d_0}$. It is interesting to note that this known expression is obtained from the solution of the diffraction problem. The derivation given here does not take into account the pressure of surface tension $P' = 2\sigma / r_0$. For water at 18°, $\sigma = 72$, and it is thus easy to see that the effects of surface tension will play a substantial part for bubbles with a radius smaller than $10^{-3}$ cm. Bubbles with a radius of less than $15 \cdot 10^{-4}$ cm * become unstable and dissolve.

The resultant velocity flux through the surface of the sphere at resonance in the case of a gas bubble in liquid will have an amplitude

$$\dot{X}_0 = 4\pi r_0^2 q_r(r_0) \simeq q_0 \frac{\lambda^2}{\pi}, \tag{6}$$

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* [The original is ambiguous at this point; $- 1.5 \cdot 10^{-4}$ or $15 \cdot 10^{-4}$ are equally plausible – editor's note].

i. e., it will be $\lambda^2/\pi$ times greater than the flux through 1 cm$^2$. From this it follows that the resonator draws in energy from the area of the plane wave front; this area is

$$S \simeq \lambda^2/\pi. \tag{7}$$

This result, as we know, was obtained by Lamb [2] for air resonators on the basis of completely different concepts.

For the amplitude of the sound pressure on the surface of the sphere we obtain the expression

$$p(r_0) = p_i(r_0) + p_s(r_0) \simeq p_0\left[1 - \frac{\rho c r_0^2\left(1 - \frac{\varkappa_1}{\varkappa}\right)}{Z_1}\left(1 + j\frac{1}{z_0}\right)\right]. \tag{8}$$

At resonance for a gas bubble in liquid we obtain

$$\frac{p(r_0)}{p_0} \simeq \frac{\varkappa_1}{\varkappa} - j\left(1 - \frac{\varkappa_1}{\varkappa}\right)\frac{1}{z_{\text{res}}} \simeq -j\frac{1}{z_{\text{res}}} = 68e^{-j\frac{\pi}{2}}, \tag{9}$$

i. e., an increase in the pressure by a factor of 68 times in comparison with the pressure in the incident wave.

From (3a) it is possible to compute the ratio between the amplitude of the pulsating vibrations and the radius of the air bubble in water at resonance:

$$\frac{a}{r_0} = \frac{1}{\varkappa z_{\text{res}}^3}\, p_0 \simeq 1.5 \cdot 10^{-5} p_0. \tag{10}$$

When $p_0 > 10^3$ bar we obtain pulsations which exceed 1.5% of the radius. On the basis of this it is clear that the application of the diffraction theory developed by Rayleigh (this derivation is based upon the boundary condition for $r = r_0$) to the vibrations of a gas bubble in water is possible only in an approximate manner for $p_0 < 10^3$ bar when it is possible to assume that $r_0 \approx$ const. For pressures of the order of $10^6$ bar and greater (these are usual pressures when working with ultrasonic waves), the theory is inapplicable; in that case the elasticity, the coupled mass and the radiation impedance will depend upon the amplitude, and the vibrating system becomes nonlinear.

The second fact which is not taken into account by the theory developed above consists of the heat-exchange conditions which obtain between the internal volume of the sphere and the surrounding medium. At the boundary of the sphere a considerable temperature gradient arises, and the vibrations of the gas bubble in the liquid cannot be considered rigorously adiabatic. This condition causes an irreversible heating and sound-absorption process to occur in the medium; the modulus of elasticity can no longer be computed from the adiabatic law. In view of this the increase in the amplitude of the pressure at resonance (by a factor of 68) which is derived above is obviously too large.

It is also of interest to investigate the oscillatory vibrations of the flexible sphere in the direction of the incident wave. The amplitude of the pressure exerted upon the surface of the sphere by the incident wave is approximately equal to

$$p_i(r_0) = p_0 e^{jkr_0 \cos\vartheta} \simeq p_0(1 + jz_0\cos\vartheta).$$

The resultant pressure force along the x axis is

$$F_x = -\int_0^{\pi} p_i(r_0)\cos\vartheta \cdot 2\pi r_0^2 \sin\vartheta\, d\vartheta = -j\frac{Sp_0}{3}z_0 = -j\omega v\rho q_0, \tag{11}$$

where $v = \frac{4}{3}\pi r_0^3$ — is the volume of the sphere.

The amplitude of the radial velocity (taking into account terms of the first and second order) in the wave which is incident upon the surface of the sphere will be equal to

$$q_{ir}(r_0) \simeq -q_0\left[j\frac{z_0}{3}+\cos\vartheta\right].$$

In the scattered wave we obtain

$$q_{sr}(r_0) \simeq -q_0\left[+j\frac{z_0}{3}\frac{\varkappa}{\varkappa_1}-j\frac{z_0}{3}-2\frac{\rho_1-\rho}{2\rho_1+\rho}\cos\vartheta\right].$$

at frequencies which are considerably lower than resonance [this equation is derived on the basis of Eqs. (2) and (3)].

The resultant amplitude of the radial velocity is

$$q_r(r_0) \simeq -q_0\left[j\frac{z_0}{3}\frac{\varkappa}{\varkappa_1}+\left(1-2\frac{\rho_1-\rho}{2\rho_1+\rho}\right)\cos\vartheta\right]. \tag{12}$$

The first term in the square brackets is caused by the pulsating vibrations; it will be smaller than the second term for the condition that $\frac{z_0\varkappa}{3\varkappa_1} < 1$, or

$$z_0 < 3\frac{\varkappa_1}{\varkappa} = z^2_{\text{res}} \tag{13}$$

We can apply this condition not only to a sphere which consists of a homogeneous substance and has a modulus of elasticity $\varkappa_1$, but also to a rigid spherical shell with an effective modulus of elasticity $\varkappa_1$ which is determined by the rigidity of the shells; in the final analysis Condition (13) is expressed in terms of the resonant frequency of the spherical shell, and this frequency depends upon the rigidity and the intrinsic mass of the shell as well as upon the coupled mass.

When Condition (13) applies, the amplitude of the vibrations of the sphere as a whole in the direction of the x axis will be approximtely equal to

$$q_x \simeq q_0\left(1-2\frac{\rho_1-\rho}{2\rho_1+\rho}\right). \tag{14}$$

When pulsating vibrations of large amplitude are present (i. e., when Condition (13) is not satisfied) the vibrations of the sphere as a whole naturally remain, but they are masked by strong pulsating vibrations.

The amplitude of the velocity of the vibrations of the sphere with respect to a medium moving with a velocity $(-q_0)$ is

$$q'_x = q_x-(-q_0) = 2\frac{\rho_1-\rho}{2\rho_1+\rho}q_0. \tag{14a}$$

For small gas bubbles in water, Condition (13) limits the applicability of Formula (14) to audio frequencies of $f < 1000$ cps. For a bulb filled with hydrogen and vibrating in air we obtain $z_{\text{res}} = \sqrt{3\frac{\varkappa_1}{\varkappa}} = \sqrt{3}$ and therefore it is necessary to guarantee that the condition $z_0 < z^2_{\text{res}} = 3$ obtains. Thus Formula (14) can be applied right up to the resonant frequency and even above this frequency.

For various relationships between $\rho_1$ and $\rho$ we obtain interesting special features of the motion of the sphere, as set forth in the table.

| Relationship between $\rho_1$ and $\rho$ | Absolute velocity $q_x$ | Relative velocity $q'_x$ |
|---|---|---|
| $\rho_1 \gg \rho$ | 0 | $+q_0$ |
| $\rho_1 = \rho$ | $-q_0$ | 0 |
| $\rho_1 \ll \rho$ | $-3q_0$ | $-2q_0$ |

The case $\rho_1 \gg \rho$ corresponds to the diffraction conditions in an immobile rigid sphere. In the case $\rho_1 = \rho$ the sphere is, as it were, a partially homogeneous (with respect to the density) liquid and vibrates together with it with a velocity ($-q_0$); the sphere has no relative velocity. This case can be realized when a spherical shell with a buoyancy equal to zero is immersed in a liquid.

The last result for ($\rho_1 \ll \rho$) is somewhat unexpected. This result can be understood by basing the explanation upon the concept involving a coupled mass which in the given case is equal to one-half of the mass of the medium displaced by the sphere.

We can assume that the amplitude of the supplementary force ($F'_x$) which is caused by the scattered wave is equal to the product of the amplitude of the relative acceleration $j\omega q'_x$ and the coupled mass $\frac{v\rho}{2}$:

$$F'_x = j\omega q'_x\left(\frac{v\rho}{2}\right) = j\omega v\rho\,\frac{\rho_1 - \rho}{2\rho_1 + \rho}\,q_0. \tag{15}$$

This force arises as a result of the relative velocity ($q'_x$) between the sphere and the medium which in turn moves at a velocity of ($-q_0$).

In the case $\rho_1 \gg \rho$ we obtain $F'_x = j\omega \frac{v\rho}{2} q_0$; this force causes motion with a relative velocity having an amplitude $q' = +q_0$, which is the inverse of the velocity of the medium ($-q_0$). In the case where $\rho_1 \ll \rho$, $F'_x = -j\omega(v\rho/2)2q_0$ and thus a coupled mass ($v\rho/2$) (which is twice as small as the mass displaced by the liquid) will move with a relative velocity having an amplitude $q'_x = -2q_0$ in the same direction as that in which the medium moves. If $\rho_1 = \rho$, then $F'_x = 0$ and there will be no relative motion between the coupled mass and the medium ($q'_x = 0$); more correctly stated, in this particular case there is no coupled mass and the sphere moves along with the medium as one whole. In this particular case we are dealing with a solid medium in which any selected spherical portion satisfies the condition $\rho_1 = \rho$, but generally speaking, this relationship can be satisfied, for example, by spherical drops of liquids which do not mix with the surrounding medium and which have a density equal to the surrounding medium, or by rigid spherical gas-filled shells which have an average density that is equal to the density of the liquid in which they are immersed.

An examination of the problem concerning the oscillatory motion of the sphere due to the effect of sound was performed for an ideal frictionless medium. It is essential to clarify how viscosity will affect this type of vibration. The amplitude of the resistive force ($f_x$) of a viscous medium to the motion of sphere can be computed [3] according to the formula

$$f_x = C\,(\pi r_0^2)\,\frac{\rho q_x'^2}{2}, \tag{16}$$

where C is a coefficient which depends upon the velocity $q'_x$. Our computations, which were made for an ideal medium, will produce results which are close to reality if the force of inertial reaction $F'_x$ is greater than the viscous force $f_x$. Let us impose the condition

$$\gamma = \frac{F'_x}{f_x} = \frac{\frac{4}{3}\omega r_0}{C q'_x} = \frac{\frac{2}{3}\omega d_0}{C q'_x} > 1. \tag{17}$$

In the case of air bubbles in water Condition (13) can be written in the more rigorous form $\frac{z_0 \kappa}{3\kappa_1} < 2$; on the basis of this condition we obtain

$$\frac{2}{3}\omega d_0 < 82 \quad \text{or} \quad f < \frac{20}{d_0}. \tag{18}$$

For Reynolds numbers which are smaller than 2 the coefficient $C = 24/\mathrm{Re}$ and $Cq'_x = 24\,\nu/d_0$; we thus obtain [on the basis of Eq. (17)] the following condition which governs the predominance of the inertial forces over the viscous forces:

$$\gamma = \frac{\omega d_0^2}{36\nu} > 1.$$

At 18° for water, $\nu \approx 0.01$; thus we obtain

$$f > \frac{0.057}{d_0^2}. \tag{17a}$$

Conditions (17) and (18) together yield a frequency range in which Formula (14) is approximately applicable for the purpose of finding the velocity of vibration for spherical bubbles of various diameters; for t = 18° we obtain

$$\begin{array}{ll} d_0 = 0.1\ \mathrm{cm} & 6 < f < 200 \\ d_0 = 0.01\ \mathrm{cm} & 600 < f < 2000 \\ d_0 = 0.003\mathrm{cm} & 6000 < f < 6700 \end{array}$$

Thus Formula (14) is applicable only for bubbles with a diameter $d_0 < 0.003$ cm and only in a narrow range of audio frequencies which varies for different bubble diameters. At higher temperatures the lower limit of the frequency range is lowered. Thus for f = 70° it is lowered by a factor of 2.5.

The condition Re < 2 is always fulfilled for bubbles of any size provided the vibration amplitudes are comparatively small ($q'_x < 0.1$ cm/sec). The situation is different for large amplitudes. Thus for a pressure amplitude of $p_0 = 2\cdot 10^6$ bar we obtain $q_0 = 13$ cm/sec and $q'_x = 26$ cm/sec. In that case for bubbles with a diameter ranging from $10^{-3}$ to $10^{-1}$ cm we utilize experimental data [3] for the values of C(Re) and obtain the result that $\gamma < 1$ only for bubbles with a diameter greater than 0.1 cm in the range of low audio frequencies. Thus, for example, for $d_0 = 0.2$ cm we have $35 < f < 100$.

For spherical shells with a large diameter which are immersed in water and for which $\rho_1 < 1$ (a positive buoyancy) we can compute $z_{\mathrm{res}}$ according to the known formula of Liav [4] for thin shells:

$$f_{\mathrm{res}} = \frac{2}{\pi d_0}\sqrt{\frac{E}{2(1-\sigma)\rho'}}, \tag{19}$$

where $\rho'$ is the density and $\sigma$ is the Poisson coefficient of the shell material. When the shell vibrates in water the increase in mass ($M = 4\pi r_0^3\rho$) due to the coupled mass of the water should be taken into account. This lowers the resonant frequency to the value

$$f'_{\mathrm{res}} = f_{\mathrm{res}}\sqrt{\frac{1}{1+\dfrac{d_0}{2h\rho'}}}. \tag{20}$$

For a steel shell with a diameter $d_0 = 50$ cm and a thickness h = 0.3 cm we obtain $f'_{\mathrm{res}} = 1540$ cps, and Condition (13) yields an upper frequency boundary of f < 2500 cps (i. e., this frequency is higher than $f'_{\mathrm{res}}$). The average density in the given case is $\rho_1 = 0.28$, and Formula (14) yields a relative velocity of $q'_x = -0.92\,q_0$ and an absolute velocity of $q_x = 1.929\,q_0$.

By establishing a device of the seismograph type inside the shell and thus measuring $q_x$, we can perform further computations on the basis of Formula (14a) to obtain the magnitude of $q_0$ and the amplitude of the acoustic pressure $p_0 = \rho c q_0$.

The motion of a spherical hydrogen-filled bulb in a gravity field will proceed in a manner analogous to the motion of a light sphere in the field of a soundwave. Its acceleration will be determined from the expression

$$\frac{dq}{dt} = -\frac{(v\rho - v\rho')g}{v\rho' + m_0 + \frac{v\rho}{2}}, \qquad (21)$$

where $\rho'$ and $\rho$ are the densities of hydrogen and air respectively, $g$ is the acceleration of gravity, $m_0$ is the mass of the shell and $\frac{v\rho}{2}$ is the coupled mass of the air; assuming that $m_0 \ll v\rho$ and $\rho' \ll \rho$, we obtain $\frac{dq}{dt} \approx -2g$ (i. e., the acceleration of the bulb at the initial instant will be twice as great as acceleration of gravity).

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Acoustics Staff of the Moscow State University

# CHRONICLE

## IU. M. SUKHAREVSKII

### (On His Fiftieth Birthday)

On September 8, 1956 one of the outstanding acoustic physicists in the USSR, a Doctor of Technical Sciences, a member of the Communist Party of the USSR, Iurii Mikhailovich Sukharevskii marked his fiftieth birthday.

Iu. M. Sukharevskii began his scientific career in 1929 as a student in the Moscow Power Institute. After finishing his studies Iurii Mikhailovich continued his scientific work in the Central Scientific Research Institute of the Communications Ministry in Moscow, where he worked in the capacity of a science associate in the acoustics laboratory. In 1938 he was invited to work in the Academy of Sciences of the USSR in the capacity of a senior science associate of a group of technical physicists in the Department of Technical Sciences of the Academy of Sciences of the USSR; from 1939 on he has been working as the director of a department of the acoustics laboratory of the P. N. Lebedev Physics Institute of the Academy of Sciences of the USSR. At the present time Iurii Mikhailovich is the director of one of the large laboratories of the Acoustics Institute of the Academy of Sciences of the USSR.

The characteristics of the scientific activity of Iu. M. Sukharevskii are the inventiveness and practicality of his research, the originality of the methods which he utilizes and his broad scientific erudition and talent as a researcher. From 1931 up to now he has published more than thirty scientific papers, including two books. In addition he has written a large number of special reports and articles. During the first period of his scientific career Iurii Mikhailovich conducted research on a number of problems in architectural acoustics and electroacoustics; the culmination of his work was his doctor's dissertation in 1940 which was devoted to new problems in the operation of sound amplification systems in the open air and indoors. Since the end of the Great Patriotic War he has been working basically in the fields of ultrasonics and the propagation of sound waves in solid and liquid media. He was the first to publish a series of papers on the reverberation of sound in the ocean.

The government of the USSR has rewarded Iu. M. Sukharevskii with Orders of the Red Banner of Labor and the Badge of Merit, as well as with other medals, for his fruitful scientific work.

Iurii Mikhailovich devotes much of his energy and creative talent to the task of developing young scientists in the field of acoustics. During the last five years he has prepared six science candidates, and he is now directing the work of a number of aspirants and student-diplomates in various universities throughout the country.

The range of interests of Iurii Mikhailovich is not limited to scientific research, and part of his time is devoted to music. He finished at Moscow Conservatory in 1931 in the class of Professor Igumnov and finished his course as an aspirant in the Conservatory; he is now a talented pianist. Iu. M. often gives concerts in the Moscow House of Scientists.

In his 50 years Iurii Mikhailovich has been full of creative energies. We wish him health and success in his fruitful scientific career.

## THE SECOND INTERNATIONAL ACOUSTICS CONGRESS

From June 17 through 24, 1956 the Second International Acoustics Congress was held in the USA; this congress was organized by the International Commission on Acoustics (President Professor R. Bolt). Approximately 1300 delegates from more than 15 countries took part in the work of the Congress – these were well known scientists, engineers and representatives of firms. The most numerous delegations were the American delegation of acoustics scientists, for whom the Congress was simultaneously the 51st annual conference of the American Acoustics Society, and the delegation from West Germany.

The delegation of Soviet acoustic scientists to the Congress, which consisted of 5 members, included corresponding member of the Academy of Sciences of the USSR L. M. Brekhovskikh (Chairman of the delegation), Academician N. N. Andreev, Doctors of Physical and Mechanical Sciences V. A. Krasilnikov and V. F. Nozdrev, and Candidate in Physical and Mechanical Sciences Iu. P. Lysanov.

The Congress held three plenary sessions on the basic trends in modern acoustics. In addition the Congress conducted 27 section sessions which were devoted to physiological acoustics, bioacoustics, the study of speech, the investigation of noises and means for their elimination, the propagation of waves in inhomogeneous media and atmospheric acoustics, the absorption of sound in gases and liquids, the propagation of sound and the effect of turbulence, the study of the radiation and scattering of sound, the propagation of sound in solid media, waves of large amplitudes and second-order effects, general acoustical physics and geophysics, musical and architectural acoustics, the recording and reproduction of sound, ultrasonics and its applications in engineering, etc.

The sessions of the Congress were held in the Massachusetts Institute of Technology (in the town of Cambridge) and at Harvard University (in the same town).

Fourteen papers were read at the plenary sessions, and these were basically in the nature of surveys. Of special interest at the plenary session on bioacoustics and the methods of eliminating noise were the papers devoted to the effect of noise upon the human organism and the paper read by L. Beranek (USA) on techniques of measuring noise.

During the following plenary session E. Meyer and V. Kühl (West Germany) read papers on architectural acoustics. Papers on the purposes, fundamental problems, achievements and trends in musical acoustics were read by A. Small and D. Martin (USA).

The session devoted to acoustical physics and ultrasonics attracted great interest. A report on the various applications of ultrasonics in engineering was made by A. Crawford (England). The paper by L. M. Brekhovskikh on the modern state of the theory of sound propagation in inhomogeneous media was heard with great interest. Interesting papers were also read by V. Lamb (England) on the molecular theory of sound propagation in gases and liquids, and by P. Bordoni (Italy) on the propagation of sound in solid bodies.

Several sections worked daily at the Congress, and each of them heard from 8 to 13 papers lasting for up to 15 minutes each. Altogether approximately 270 papers were read.

The section on the propagation of waves in inhomogeneous media and atmospheric acoustics heard papers by E. Skudridge (USA) on the propagation of sound waves of any arbitrary frequency in a medium containing random inhomogeneities of arbitrary dimensions, by D. Preedmore-Brown (USA) on the investigation of a sound field in the shadow zone formed by the wind velocity gradient. R. Lyon (USA) read a paper on the diffusion of sound waves in turbulent atmosphere. A number of papers in this section were devoted to the problems of sound generation which is caused by the very rapid movement of bodies and to the interaction of sound waves with high-velocity streams (i. e., to problems which are such that acoustics becomes directly associated with modern aerodynamics).

In the section devoted to the radiation and scattering of sound H. Marsh (USA) reported on an accurate solution which he had obtained for the problem involving the scattering of sound on an uneven surface. The paper by the Canadian scientist H. Hips was devoted to the investigation of sound scattering on a surface whose impedance is a random function of a point on the surface. R. Urik and R. Hoover (USA) reported on the results of their experimental investigation of the scattering of sonar sound pulses on the surface of the ocean. Other papers examined the problems of radiation from a source located in the vicinity of a reflecting plane, the radiation efficiency of a chain of point-source radiators with a periodic phase variation, etc.

Two section sessions were devoted to problems of sound absorption in gases and liquids. A paper by T. Litovitz (USA) was devoted to the theory of thermal relaxation when sound waves are propagated in a liquid. The paper by T. Hashimoto and U. Kikuchi (Japan) analyzed the results obtained from measuring ultrasonic absorption in sea water of frequencies of from 10 to 400 kc. P. Mareens (Belgium) reported on the results of his experimental investigation of the absorption of sound in wide tubes and in capillary tubes at various pressures. A group of French scientists reported on the results of measurements of the ultrasonic velocity in various gases at high pressures (up to 1200 atmos at various temperatures and frequencies (as well as in the vicinity of the critical point).

A significant place in the section on the propagation of waves and the effect of turbulence was occupied by the problem of the formation of sound in a boundary layer when bodies move at great velocities (including supersonic velocities).

Four sections of the Congress were devoted to problems involving the investigation and elimination of various types of noise. A number of papers reported on the investigation of noise under factory conditions and discussed measures for lowering the noise. In particular, it was suggested that the noise can be lowered by shielding the sound source with sound-absorbent panels; in a number of cases this made it possible to lower the noise level by 10-20 db even in the near field. Other authors suggested that noise should be muffled by means of resonant filters. At another section session the Dutch scientists D. Ein, M. Kastelein and K. Kostin reported on certain most interesting results of investigations of the creation of sound insulating structures for living quarters and on the standards adopted in Holland for the sound insulation of living quarters, in addition to a program for further investigations in this direction. At the same section a number of papers were read which were devoted to the problem of creating effective sound-absorbent materials and sound insulating structures, as well as to methods of measuring noise in rooms. A large number of reports at the two other sections were devoted to the investigation of noise created by various machines, airplanes (including jet planes), city noise, and to the various measures which are used to combat them.

Ultrasonics and its applications in medicine, chemistry, in industry for the purposes of defectoscopy, the processing of solid materials, the reaction upon melts in metallurgy, etc., occupied an important place in the work of the Congress.

In the sections on physiological and biological acoustics papers were presented on the physiology of hearing, the perception of sound, on investigations of the auditory threshold of a human being at various frequencies in water. The study of speech and the equipment for its analysis and synthesis was concentrated in two corresponding sections.

The members of the Soviet delegation presented papers devoted to the focusing of sound waves by inhomogeneous media (L. M. Brekhovskikh), to the propagation of waves in media with random inhomogeneities (V. A. Krasil'nikov), to certain problems which border between acoustics and molecular physics (V. F. Nozdrev), to the scattering of waves on inhomogeneous surfaces (Iu. P. Lysanov). In addition, papers by other Soviet scientists were read at the Congress, in particular, papers by G. Gershuni on new methods for measuring human hearing, by L. D. Rozenberg, A. S. Bebchuk and L. O. Makrov on the mechanism of damage to surface films in an ultrasonic field, by S. M. Rytov and M. L. Levin on the transition to a geometric approximation in the theory of elasticity, by B. Z. Tartakovskii on ultrasonic transition layers, by L. A. Chernov on the correlation properties of a sound field in a medium with random inhomogeneities, by I. G. Mikhailov on the absorption of ultrasonic waves in liquid, by V. A. Zverev on a new method for measuring ultrasonic dispersion, by A. N. Leporskii on the scattering of sound on an uneven surface.

Mention should be made of the great assistance accorded to our delegation by a group of young scientific workers of the Technological Institute – M. Halle, A. Trilling, D. Preedmore-Brown, R. Lyon, and D. Leeklider – who read a number of papers and assisted in translation.

Besides presenting papers, the members of our delegation took part in the supervision of the Congress. Thus, for example, Academician N. N. Andreev was chairman of the section on large-amplitude waves and second-order effects.

In addition to participating in the sessions of the Congress the members of our delegation became acquainted with the scientific work and the equipment of a number of acoustic laboratories in the USA. Thus in Princeton they examined the acoustics laboratory of the RCA firm which was directed by H. Olsen, where they became acquainted with the work being done on the recording and reproduction of sound, on the development of loud speakers, microphones, mechanical filters and on the investigation of stereophonic systems. A very large and very well equipped sound-insulated chamber is available in the laboratory for acoustical measurement.

In Cambridge the delegation visited the acoustics laboratory of the Massachusetts Institute of Technology – one of the largest scientific-research and study centers of the USA. The basic scientific interests of the laboratory are concentrated in the fields of investigating the sound field in the shadow zone, the scattering of waves in a statistically inhomogeneous medium in a wave duct, the scattering of waves when they are propagated above an inhomogeneous surface, the scattering of sound by sound, the absorption of sound waves in gases at high temperatures, etc. A number of well-known scientists in the field of acoustics work in the laboratory – O. Mavardi, U. Ingord and others. There are very many young science associates working in the laboratory, and this is generally characteristic at the present time of American scientific-research institutes.

In Brown University acoustics laboratory (Providence, R. I.) the delegation was shown very interesting work which was in the process of being done on the investigation of sound absorption in metals at low temperatures (both in the superconductor state and in the vicinity of this state). In this same laboratory experiments are being conducted in hydroacoustics, nonlinear acoustics and physiological acoustics.

The acoustics laboratory of Columbia University in New York (which is directed by S. Garris) left a very good impression. Here a large number of practical problems in acoustics are being investigated – for example, the formation of sound in the vicinity of surfaces which move at a supersonic velocity, the absorption of low-frequency sound in gases, etc.

The members of the delegation became acquainted with the work that is being done in the field of molecular acoustics (absorption, dispersion, relaxation processes in gases and liquids) in the acoustics laboratory of the Catholic University in Washington. In the acoustics laboratory of Harvard University the delegation was shown a correlator – a device for automatically finding the correlation coefficient. In this same laboratory an international exhibition of acoustics equipment was organized during the period of the Congress; exhibits were presented by firms of the USA, West Germany, England, Denmark, Canada and several other countries. Equipment for the generation and analysis of sound at various frequencies was demonstrated, as well as equipment for the recording and analysis of speech and apparatus for sound measurement and reception (microphones, hydrophones).

The very broad scope of scientific research which is being conducted in the USA in the fields of theoretical as well as experimental acoustics left a profound impression. The number of acoustics scientists is very great. A great deal of attention is being devoted to the application of research results to practical problems. Especially evident is the fact that all universities which the delegation was able to visit have well-equipped acoustics laboratories. It is precisely this presence of acoustics laboratories in universities which makes it possible to conduct scientific-research work of very wide scope in the field of acoustics. Also worthy of mention is the fact that standard instruments, individual parts, etc., which may be needed in carrying out research work and which, as a rule, cannot be planned for in advance can be easily bought in specialized stores; this speeds up the completion of experimental work to an enormous degree.

It is interesting to note that many American scientists in the field of acoustics prefer to perform the basic portion of their scientific work on the basis of contracts arrived at with various commerical firms or other institutions. This frees the scientists from the extra bureaucracy and red tape which obtains when work is done for state institutions.

The Department of the Navy provides a great deal of support for the development of acoustics in the USA; a large amount of work is done on the basis of contracts with the Navy. According to one American scientist, a representative of the Navy announced that the Naval Department would pay for any work in acoustics even if it was of a completely abstract nature.

Very characteristic for the USA is the active participation of acoustical physicists in the field of building and the very wide application of sound-absorbent materials. We inspected a number of radio broadcasting studios, television studios, the large Radio City Music Hall which accommodates 6000 people, the assembly halls in the United Nations Organization Building (all in New York). All of the rooms are distinguished by their good acoustical properties; these properties are achieved because of the great experience of the research workers who assist in their design and because of the application of sound-absorbent materials. Special scattering structures are also utilized, and these are constructed, for example, in the form of wooden beam systems which are disposed vertically along the side walls.

The many meetings between the members of our delegation and the foreign scientists during the period of the Congress made it possible to establish personal contacts with many scientists of the USA, France, West Germany, Italy and other countries, and to exchange literature and agree to exchange scientific correspondence. The delegation encountered a very warm reception from the organizers and participants at the Congress. First of all it should be noted that the President of the International Commission on Acoustics, Professor R. Bolt, exhibited an extremely friendly attitude toward our delegation; it was with great hospitality that he accorded the delegation a great deal of assistance during its stay in the USA. Other American scientists – L. Beranek, S. Garris, U. Ingord – gladly acquainted the members of the delegation with the living and working conditions of American scientists and told them of their scientific work and plans for the future. The scientific workers of the universities always made every effort to fulfill any and all wishes of the delegation with regard to the inspection of laboratories. For example, when it became known that the delegation would not be able to visit the Brown University laboratory due to lack of time, the scientists of that university arranged to show the delegation the laboratory on a nonworking day. During the tour of the laboratory not only the directors were present but also all of the research workers who were working on specific projects; this permitted the members of the delegation to become acquainted with the experimental work in the actual process of its fulfillment. In turn, the Soviet delegation arranged a reception in honor of the organizers of the Congress and the acoustics scientists of other countries; this reception was enjoyed by all in a friendly and informal atmosphere. 150 copies of the Journal of Acoustics were given to the organizing committee of the Congress, and these included many of the Soviet papers which were presented at the Congress.

In conversations with members of the delegation many of the scientists expressed their desire for the future improvement of scientific contacts with the Soviet Union, and these wishes were fully shared by us as well. Foreign scientists evidenced a deep interest in our scientific achievements. It is sufficient to say that a number of Soviet scientific journals, for example, the Journal of Experimental and Theoretical Physics and the Proceedings of the Academy of Science of the USSR (Physics Division) are translated in full in English in the USA. In the immediate future it is planned to translate the Journal of Acoustics. On the return journey from the USA the delegation visited two acoustics laboratories located near Paris at the invitation of a number of French scientists.

In the laboratory of Physiological Acoustics of the National Research Institute a number of interesting experiments are being performed in agronomy in the direction of finding acoustical means of combating agricultural crop destroyers (in particular, to combat locusts). A number of preliminary results had been obtained, and these showed the practicability of the selected direction of research.

Interesting work on the absorption, dispersion and relaxation of sound in gases and liquids at high pressures was shown to us in the laboratory of High Pressures of the Scientific Research National Center.

The International Commision on Acoustics decided to conduct the next (third) International Acoustics Congress in 1959 in the city of Stuttgart (West Germany). Together with this the International Commission on Acoustics proposed that a conference on the propagation of waves in inhomogeneous media be conducted in our country within the next two years.

## THE FIFTH CONFERENCE ON PROBLEMS OF SPEECH PERCEPTION

On May 9 and 10 of this year the Fifth Conference of the Section on the Investigation of Speech, attached to the Acoustics Commission of the Division of Physical and Mathematical Sciences of the Academy of Sciences of the USSR was conducted in Leningrad; this conference was devoted to problems of speech perception.

At the first session papers were read on the physiological and psychological features of speech perception. The fundamental problems discussed were those concerning the dependence of speech perception upon its physical features and upon the system lingual values, the problem of the relationship between processes of a peripheral and central nature during the perception of speech, and the problems involved in the sensory responses and the thought processes which occur.

L. A. Chistovich (Leningrad I. P. Pavlov Institute of Physiology of the Academy of Sciences of the USSR) reported "On the special features of the perception of speech sounds," after having examined the physiological features of the process of the perception of individual speech sounds on the basis of a series of experiments. The reporter, as a speech physiologist, was interested in the study of the mechanisms governing the dependence of the recognition of each individual speech sound upon the system of phonemes of a given language. Physically this means that speech sounds are perceived on the basis of their formant characteristics, since phonemes are physically counteracted by the more or less constant formant bands in their spectra. Here L. A. Chistovich examines each phoneme of a given language as a conditional signal; its reception is examined as a conditional reaction, and other phonemes which are possible in the given language are examined as differentiated stimuli which are associated with other reactions. The sound of speech is a complex conditional signal which can be considerably generalized with respect to each individual sound element (i. e., with respect to each parameter of the sound).

I. K. Samoilova (Moscow, Institute of Biological Physics of the Academy of Sciences of the USSR) in her paper titled "The masking of speech sounds at high levels of intensity" indicated the possibility of masking a weak tone by a subsequent strong tone, depending upon the relationship between their intensity, duration, time separation and frequency. This phenomenon, in the opinion of Samoilova, cannot be explained by processes which go on at the periphery of the auditory analyzer; it must be associated with the central sectors of the auditory analyzer.

E. Ia. Voitinskii (Leningrad I. P. Pavlov Institute of the Academy of Sciences of the USSR) reported "On certain regularities in the perception of complex sounds under selective conditions" in the absence of a standard sound. Experiments were conducted with complex sounds which were obtained by means of amplitude modulation, and with pure tones. Experiments showed that the boundaries of discrimination are determined not only by the physical structure of the sound but also by the conditions governing its discrimination (in particular, by the number of stimulus sources from which the selection is made).

V. A. Artemov (Moscow, Laboratory of Experimental Phonetics and Speech Psychology 1 MGPIIIa*
in his paper titled "Speech perception" noted that in general one and the same circuit obtains for all cases of oral speech perception. The source of information (the person speaking or the tape recorder) emits information which by its physical properties (frequency, overtones and time-varying amplitudes) and by its lingual values reacts upon the sensory organs and consciousness of the listener. The sensory organs are special transmitters which transform the oral speech into nerve signals which are coded according to the phonetic, lexical, grammatical and stylistic system of the given language. The articulation apparatus of a speaking human being serves to solve the problems of the production of speech and its coding according to the system of the language. The auditory and articulation

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* [A division of, Moscow State Pedagogical Institute].

apparatus (and to a known degree the visual apparatus) of the listening perion serves as the receiver which decodes both the organized nerve process in the audible or visual speech. During this time the speech is filled with thought content, is perceived and is understood by the person receiving it. The adequacy of the perception and understanding of speech becomes evident in the actions (the responses) of the person perceiving the speech.

Not only the physical properties of speech serve as the objective basis in the perception of speech, but also its lingual values. The physical properties of speech, which are recognized by a human being become its perceived properties: the acoustical content, syllables, words, phrases (with one or another intonation) and the periods of speech. These lingual units in speech are organized according to its phonetic, lexical, grammatical and stylistic system. Thus in perceiving speech a person responds to it as to a system of secondary-signal stimuli (i. e., as to a system of lingual values which are surrounded by the sound fabric of the language).

The second session of the conference was devoted to the pathology of sound perception.

V. F. Undrits (Leningrad, the first I. P. Pavlov Leningrad Medical Institute) made a report on "The perception of speech when there is peripheral damage of the auditory apparatus." The reporter indicated the fact that the precise functional characteristic of the sound analyzer is of substantial importance in clinical work, in prophylaxis with regard to deafness, in sordine-logopedics, in selection hearing aids, in the selecting the type of prophylaxis, etc. Here the study of the communicative functions of speech as a process of intercourse by means of language and its perception plays an outstanding part.

N. N. Traugott (Leningrad, I. M. Sechenov Institute of Evolutionary Physiology of the Academy of Sciences of the USSR) read a paper on the theme "The problem of the special features of the auditory function when the cortex section of the auditory analyzer is damaged."

V. K. Orfinskaia (Leningrad, A. I. Gertsen State Pedagogical Institute) reported on "Types of acoustical alalia."

The third session of the conference heard papers on the theme of speech and auditory speech reception in children during the first years of their lives.

A. I. Bronshtein, E. P. Petrova, A. M. Bruskina and A. G. Kamenetskaia (Leningrad, I. M. Sechenov Institute of Evolutionary Physiology of the Academy of Sciences of the USSR and the Third Hospital Clinic of the Leningrad State Pediatric Medical Institute) reported on the "Investigation of hearing in new-borns and in children of early breast-feeding age;"

A. M. Maruseva (Leningrad, I. P. Pavlov Institute of Physiology of the Academy of Sciences of the USSR) reported on the "Measurements of hearing in children during the first years of their lives;"

M. M. Kol'tsova (Leningrad, I. P. Pavlov Institute of Physiology of the Academy of Sciences of the USSR) reported on "The physiological study of speech."

The conference unanimously adopted a resolution in which the outstanding fruits of the many-sided study of speech by physicists, biophysicists, physiologists, psychologists, linguists, mathematicians and communications engineers is sharply underlined. The resolution notes certain progress in the study of speech. It indicates the necessity of expanding the methods of mathematical (statistical) analysis of speech and also the necessity of giving aid in every way to the study of not only the effect of the physical properties and perception qualities of speech upon the process of intercourse by means of language, but also the effect of its lingual values – phonetic, lexical, grammatic and stylistic.

The conference once again affirmed the necessity of publishing the special organ of the section – the journal "Speech."

V. A. Artemov

# THE JOURNAL OF ACOUSTICS OF THE ACADEMY OF SCIENCES OF THE USSR IS PUBLISHED IN THE USA

**The American Institute of Physics which has recently begun to publish the Journal of Experimental and Theoretical Physics of the Academy of Sciences of the USSR in English, announced in Issue No. 4 of Vol. II of this journal that it would publish the Journal of Acoustics of the Academy of Sciences of the USSR in English. The subscription price is $ 20 per volume for the USA and $ 22 for other countries. Publishing will begin with the first volume of the Journal.**

# LETTERS TO THE EDITOR

## THE POSSIBILITY OF ABSOLUTELY CALIBRATING SOUND RADIATORS AND RECEIVERS ACCORDING TO THE RADIATION PRESSURE, WITHOUT THE UTILIZATION OF A RADIOMETER

V. A. Zverev

As we know, the ponderomotive effect of sound (this effect is caused by radiation pressure upon the absorbing wall) can be expressed by the formula

$$P = \frac{p^2}{2\rho c^2}. \tag{1}$$

where p is the amplitude of the sound pressure, $\rho$ is the density of the medium, c is the velocity of sound. As we have indicated earlier, [1], Eq. (1) makes it possible to perform the absolute calibration of both radiators and receivers of sound when modulation of the radiation pressure is utilized (as proposed by Pumper [2]). Let us direct sound of frequency $\omega$ modulated by a frequency $\Omega$ toward the receiver being calibrated. Under these conditions the radiation pressure will have a component with the frequency $\Omega$; this component is equal to

$$P_{\Omega} = \frac{m p_{\omega}^2}{\rho c^2}, \tag{2}$$

where m is the depth of modulation.

Let the receiver receive both frequencies. For a linear receiver we have

$$P_{\Omega} = \alpha_{\Omega} V_{\Omega}, \quad P_{\omega} = \alpha_{\omega} V_{\omega}, \tag{3}$$

where $V_{\Omega}\omega$ are the voltages at the output of the receiver or amplifier.

From Eqs. (3) and (2) we have

$$p_{\omega} = \frac{\rho c^2}{m} \frac{\alpha_{\Omega}}{\alpha_{\omega}} \frac{V_{\Omega}}{V_{\omega}}. \tag{4}$$

In order to determine $\alpha_{\Omega}/\alpha_{\omega}$ a wave of an auxiliary frequency $\omega_0$ modulated by the frequencies $\omega$ and $\Omega$ (simultaneous modulation is permissable) is applied to the receiver. It follows from Eq. (2) that under these conditions equal pressures correspond to equal modulation depths. The ratio $\alpha_{\Omega}/\alpha_{\omega}$. is determined according to the ratio between the voltages $V_{\omega}$ and $V_{\Omega}$ which are obtained during this experiment. Thus all of the quantities which appear in Eq. (4) can be determined experimentally, and this means that absolute calibration can be achieved.

Certain remarks should be added to the analysis made above. In the first place, when the sound intensity is small the radiation pressure is small; on this basis one might conclude that the method analyzed above is inapplicable in such cases. However, this is not so. The matter involved here is that in order to realize the measurements, only waves of fixed frequencies $\omega$ and $\Omega$ are required. This can be realized for waves which have an intensity of any arbitrary smallness, provided that a sufficiently narrowband filter is utilized. In the second place, Formula (1) which lies at the foundation of the method has been refined more than once [3, 4, 5]. Nevertheless, this formula is utilized for the purposes of absolute calibration. The difference between the method developed above and the usual method consists of the fact that when the modulation frequency is sufficiently high the effect of acoustic wind can be neglected. By the same token this method provides certain possibilities in terms of the experimental investigation of acoustic wind.

In conclusion let us note that the calibration of radiators is conducted in an analogous manner. Here the "recoil" experienced by the surface of the radiator when the wave is emitted plays the part of the radiation pressure. This phenomenon was experimentally verified by us, and we investigated the resonant curves of radiators by means of it [and by applying Eq. (2)]. For this purpose waves of a frequency $\omega$ modulated by a frequency $\Omega$ were applied to the radiator. A receiver tuned to the frequency $\Omega$ was placed in the medium outside the ray $\omega$. When $\omega$ was varied the voltage at the output of the receiver varied in accordance with the resonant curve of the radiator.

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# INVESTIGATION OF THE ULTRASONIC PROPERTIES OF ETHYL ACETATE IN THE CRITICAL REGION

### V. F. Nozdrev and V. D. Sobolev

**The study of the properties of matter in the critical region is at the present time a very practical one, both for the creation of a theory of the liquid state and for the solution of some most important problems of modern heat-power engineering.**

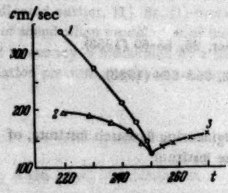

**Fig. 1. Velocity of ultrasonic waves in the critical region of ethyl acetate: 1) liquid, 2) saturated vapor, 3) reheated vapor.**

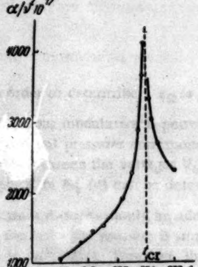

**Fig. 2. Absorption of ultrasonic waves in ethyl acetate in the critical region ($\nu$ = 5.1 megacycles).**

**In this regard ultrasonic investigations of the critical region of a substance are of considerable interest, since they permit the determination of the specific heat which is associated with the internal and external degrees of freedom of the molecules, the instanteous compressibility, the relaxation time, etc.**

**However, the existing ultrasonic investigations of the critical region are associated with substances having low values of the critical parameters $T_{cr}$ and $P_{cr}$, and only individual papers have been devoted to the investigation of the properties of liquids with high values of the parameters $T_{cr}$ and $P_{cr}$.**

**As far as measurements of ultrasonic absorption along the saturation line in the critical region are concerned, there is only one paper [1] devoted to absorption measurements in the critical region of sulfur hexafluoride ($T_{cr}$ = 45.55°, $P_{cr}$ = 36 atmos). This can be explained by the difficulties in experimental procedure which are encountered in making absorption measurements under conditions of high temperatures and pressures (a number of authors have turned their attention to this [2, 3]).**

**In order to solve the problem that has been posed we conducted measurements of the velocity and absorption of ultrasonic waves in the critical region of ethyl acetate by means of the pulse method.**

**The velocity measurements (Fig. 1) show that the ultrasonic velocity in the liquid phase of ethyl acetate (1) decreases with an increase in temperature and has a minimum at the critical point; in the reheated vapors (3) the ultrasonic velocity increases slowly with an increase in temperature. On the basis of the measurements made on the ultrasonic velocity in the liquid phase and on the basis of data on the ultrasonic velocity in the saturated vapors (2) [4] (see Fig. 1) we established the fact that the "rectilinear diameter" rule is fulfilled**

for the acoustic stiffness of the liquid and vapor phases of the ethyl acetate (this rule was theoretically proven by Nozdrev [3]).

The ultrasonic velocity at the critical point, as determined on the basis of the "rectilinear diameter" rule, proved to be equal to 127 m/sec.

In measuring the absorption we utilized the single-fixed-distance pulse method developed by Iakovlev [5]; this method made it possible to determine experimentally the absorption in those temperature ranges where it is sufficient to take into account only the variations of the acoustic impedance of the medium and the coefficient of reflection from the reflector. The upper boundary of these temperature ranges is the boiling point of the liquid in most cases. For large temperatures it is necessary to take into account the variation in the properties of the quartz radiator which is located in the working medium.

V. D. Sobolev showed [6] that at still higher temperatures the absorption coefficient is determined according to the formula:

$$\alpha_2 = \alpha_1 + \frac{\Delta A}{20 l \log e} + \frac{1}{l \log e} \log\left[\left(\frac{1-\beta t_2}{1-\beta t_1}\right)^2 \frac{k'' \rho' v'}{k' \rho'' v''}\right],$$

where $\alpha_1$ is the absorption coefficient at the temperature $t_1$, $l$ is the acoustical path length, $\Delta A$ is the variation in the amplitude of the sound pressure in decibels, $\beta$ is the temperature coefficient of the quartz piezomodulus, k' and k" are the coefficients of reflection from the steel reflector at the temperatures $t_1$ and $t_2$, c'v' and c"v" are the acoustic impedances of the medium at the temperatures $t_1$ and $t_2$.

Formula (1) which we have obtained takes into account the temperature variation not only of the acoustic impedance of the medium, but also of the quartz parameters; it thus makes it possible to determine the ultrasonic absorption coefficient up to temperatures of the order of 300 degrees (i. e., it makes it possible to perform measurements of the absorption in the critical region for the majority of liquids while retaining the accuracy which is assured by the pulse method).

We used the indicated method to measure the absorption of ultrasonic waves along the saturation line in the critical region of ethyl acetate in the frequency range of 5-9 megacycles; under these conditions the absorption measurements at a frequency of 5 megacycles also encompass the region of reheated vapors. It was these measurements which first showed that the pulse method can be applied to absorption measurements not only in a liquid phase but also in vapors at high pressures.

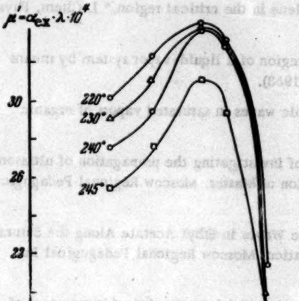

Fig. 3. Absorption of ultrasonic waves in the liquid phase of ethyl acetate in the critical region.

The result of measuring the absorption at a frequency of 5 megacycles is shown in Fig. 2 in the form of the graphical dependence of the quantity $\alpha/\nu^2$ upon temperature. By examining the curve which is plotted there we can deduce that in the liquid phase in the critical region the absorption increases with an increase in temperature and has an anomalously high value at the critical point; in reheated vapors of ethyl acetate the absorption is decreased with an increase in temperature.

The dependence of ultrasonic absorption upon temperature in ethyl acetate which we have found here is analogous to the dependence of absorption upon temperature in sulfur hexafluoride $SF_6$, notwithstanding the evident difference between the physical and chemical properties of the two substances. The fact that the derivative $d\alpha/dT$ passes through a sharp maximum at the critical point is evidently a common property of these substances and can be viewed as one of the characteristics of the critical state.

On the basis of measurements of ultrasonic absorption in the critical region of the liquid phase of ethyl acetate we determined the dependence of the absorption per wavelength ($\mu'$) upon frequency; this function is shown in Fig. 3. By examining this graph it is possible to draw the conclusion that the relaxation process which occurs at lower temperatures [6, 7] is retained in the critical region of ethyl acetate. This is verified by the fact that the curves for the dependence of the excess absorption $\mu'$ upon frequency in the critical region pass through sharply expressed maximums.

The relaxation frequency in the critical region of ethyl acetate is approximately equal to 7.5 megacycles; this corresponds to a relaxation time of $\tau = 2.12 \cdot 10^{-8}$ sec.

The discovery of the relaxation process in the critical region of ethyl acetate makes it possible to establish one of the causes for the discrepancy between data on the determination of the per unit specific heat at constant volume $C_v$ by means of the thermodynamic method and data determined from acoustical measurements [8]. The fact that values of $C_v$ found by direct calorimetric measurements exceed values of $C_v$ which are obtained from data on the ultrasonic velocity may be caused by the fact that the variation in pressure in the ultrasonic wave when the relaxation process is present becomes substantially dependent upon time. This causes the derivative $(dP/dv)_T$ to be dependent upon time, and thus the value of $C_v$ at the critical point decreases.

It is important to underline the fact that the discovery of a full relaxation region in the vicinity of the critical point permits us to perform the computation of the specific heat which is associated with the internal degrees of freedom of the molecules ($C_i$) as well as the activation energy of the relaxation process $E_a$.

The specific heat $C_i$ which was computed for the 220° isotherm proved to be equal to 0.2 cal/deg·mole; at still higher temperatures it becomes impossible to compute $C_i$ because of the absence of data on the shear viscosity as well as of other data which are necessary for the computation.

The average value of the activation energy of the relaxationprocess in the critical region of ethyl acetate is 1.9 kcal/mole; this is close to the value of the activation energy of the relaxation process in acetic acid [9].

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